leading proponent or champion of sociobiology, someone whose feelings about the field are by definition positive.

Wilson is certainly not unhappy or *strongly displeased* with this potential unified theory, nor is he merely long-suffering or *resigned* to it. Similarly, he is not *unbiased* and *objective* about it; he actively involves himself in arguing the case for sociobiology. Thus, you can eliminate Choices C, D, and E. But how do you decide between the two positive terms, *enthusiasm* and *optimism*, Choice A and Choice B? To decide between them, you must look carefully at the adjectives modifying them. Is Wilson's enthusiasm unqualified or *unconditional?* You may think so, but look again. The opening sentence states a basic condition that must be met before there can be a unified science of sociobiology: the same parameters and mathematical principles must be used to analyze insect and vertebrate societies. Though a proponent of sociobiology, Wilson is first and foremost a scientist, one who tests hypotheses and comes to logical conclusions about them. *Unconditional enthusiasm* seems to overstate his attitude.

Choice A appears incorrect. What of Choice B? Is Wilson's optimism *cautious* or guarded? Yes. According to the passage, Wilson is aware that specialists may well find fault with the sociobiologist's conclusions; the passage uses terms that convey values, first the negative "superficial, even unscientifically glib" to suggest the specialist's negative attitude toward sociobiology, then the positive "deliberate" to convey Wilson's own more positive response. The correct answer is Choice B.

TACTIC 6 When Asked About Specific Details in the Passage, Spot Key Words in the Question and Scan the Passage to Find Them (or Their Synonyms)

In developing the main idea of a passage, a writer will make statements to support his or her point. To answer questions about such supporting details, you *must* find a word or group of words in the passage supporting your choice of answer. The words "according to the passage" or "according to the author" should focus your attention on what the passage explicitly states. Do not be misled into choosing an answer (even one that makes good sense) if you cannot find it supported by the text.

Detail questions often ask about a particular phrase or line. In such cases, use the following technique:

1. Look for key words (nouns or verbs) in the answer choices.
2. Scroll through the passage, looking for those key words or their synonyms. (This is *scanning*. It is what you do when you look up someone's number in the phone directory.)
3. When you find a key word or its synonym in a sentence, reread that sentence to make sure the test makers haven't used the original wording to mislead you.

Read the following brief passage and apply this tactic.

What is involved in the process of visual recognition? First, like computer data, visual memories of an object must be stored; then, a mechanism must exist for them to be retrieved. But how does this process work? The eye triggers the nerves into action. This neural activity constructs a picture in the brain's memory system, an internal image of the object observed. When the eye once again confronts that object, the object is compared with its internal image; if the two images match, recognition takes place.

Among psychologists, the question as to whether visual recognition is a parallel, single-step operation or a sequential, step-by-step one is the subject of much debate. Gestalt psychologists contend that objects are perceived as wholes in a parallel operation: the internal image is matched with the retinal impression in one single step. Psychologists of other schools, however, suggest the opposite, maintaining that the individual features of an object are matched serially with the features of its internal image. Some experiments have demonstrated that the more well-known an object is, the more holistic its internal image becomes, and the more parallel the process of recognition tends to be. Nonetheless, the bulk of the evidence appears to uphold the serial hypothesis, at least for simple objects that are relatively unfamiliar to the viewer.

Now look at the following question on a specific detail in the passage.

EXAMPLE

According to the passage, psychologists of the Gestalt school assume which of the following about the process of visual recognition?

Select *all* that apply.

- A The image an object makes on the retina is exactly the same as its internal image.
- B The mind recognizes a given object as a whole; it has no need to analyze the object's constituent parts individually.
- C The process of matching an object with its internal image takes place in a single step.

You can arrive at the correct answer to this question by elimination.

First, quickly scan the passage looking for the key word *Gestalt*. The sentence mentioning Gestalt psychologists states they maintain that objects are recognized as wholes in a parallel procedure. The sentence immediately preceding defines a parallel procedure as one that takes only one step.

Now examine the statements. Do Gestalt psychologists maintain that an object's retinal image is exactly the same as its internal image? Statement A is unsupported by the passage.

Statement B is supported by the passage: lines 11–12 indicate that Gestalt psychologists believe objects are recognized as wholes.

Statement C is supported by the passage: lines 12–13 indicate that Gestalt psychologists believe matching is a parallel process that occurs in one step.

Choices A, B, and C are all correct.

Note how necessary it is to point to specific lines in the passage when you answer questions on specific details.

TACTIC 7 When Asked to Make Inferences, Base Your Answers on What the Passage Implies, Not What It States Directly

In *Language in Thought and Action*, S. I. Hayakawa defines an inference as "a statement about the unknown made on the basis of the known."

Inference questions require you to use your own judgment. You must not take anything directly stated by the author as an inference. Instead, you must look for clues in the passage that you can use in deriving your own conclusion. You should choose as your answer a statement that is a logical development of the information the author has provided.

Try this relatively easy inference question, based on the previous passage about visual recognition.

EXAMPLE

One can infer from the passage that, in visual recognition, the process of matching

- Ⓐ requires neural inactivity
- Ⓑ cannot take place if an attribute of a familiar object has been altered in some way
- Ⓒ cannot occur when the observer looks at an object for the very first time
- Ⓓ has now been proven to necessitate both serial and parallel processes
- Ⓔ can only occur when the brain receives a retinal image as a single unit

Go through the answer choices, eliminating any choices that obviously contradict what the passage states or implies. Remember that in answering inference questions you must go beyond the obvious, beyond what the authors explicitly state, to look for logical implications of what they say.

Choice A is incorrect. Nothing in the passage suggests that the matching process requires or demands neural inactivity. Rather, the entire process of visual recognition, including the matching of images, requires neural *activity*.

Choice D is incorrect. It is clear from the passage that the matching process is not fully understood; nothing yet has been absolutely *proven*. The weight of the evidence *seems* to support the serial hypothesis, but controversy still surrounds the entire question.

Choice E is incorrect. It can be eliminated because it directly contradicts information in the passage stating that recognition most likely is a serial or step-by-step process rather than a parallel one receiving an image as a single unit.

Choices B and C are left. Which is a possible inference? Choice C seems a possible inference. Although the author never says so, it seems logical that you could not match an object if you had never seen it before. After all, if you had never seen the object before, you would have no prior internal image of it and would have nothing with which to match it. What of Choice B? Nothing in the passage mentions altering any attributes or features of a familiar object. Therefore, *on the basis of the passage* you have no way to deduce whether matching would or would not be possible if such a change took place. There is not enough information in the passage to justify Choice B as an inference. The correct answer is Choice C.

Another, more difficult inference question is based on the previous excerpt reviewing Wilson's *Sociobiology*. Review the passage briefly and see how you do with a question that very few of the examinees would have answered correctly.

According to Wilson, only when we are able to apply the same parameters and mathematical principles to weighing both troops of rhesus macaques and termite colonies will a unified science of sociobiology
Line finally exist. While recognizing that many of his colleagues question such
(5) an outcome, Wilson, one of sociobiology's leading proponents, finds himself simultaneously more and more struck by the functional similarities that characterize both insect and vertebrate societies and less concerned with the structural differences that divide them to such an apparently irreconcilable degree. Thus, he freely compares termites and macaques,
(10) pointing out numerous likenesses between them. Both societies are territorial: they occupy a particular home range, which they defend against intruders. Likewise, both are cooperative: members organize themselves into working groups that observe a clearly-defined division of labor. In addition, members of both groups can convey to each other a range of
(15) basic emotions and personal information: animosity, fright, hunger, rank within a particular caste, and ability to reproduce. Wilson readily concedes that, from a specialist's perspective, such a likeness may at first appear superficial, even unscientifically glib. Nonetheless, in this eminent scholar's judgment, "it is out of such deliberate oversimplification that the
(20) beginnings of a general theory are made."

EXAMPLE

In analyzing insect and vertebrate societies, the passage suggests which of the following?

- Ⓐ A clearly-defined division of labor is a distinguishing feature of most insect and vertebrate societies.
- Ⓑ The caste structures of insect and vertebrate societies share certain likenesses.
- Ⓒ Most insect and vertebrate societies utilize cooperative groups to hold and defend their home range.
- Ⓓ The system of communication employed by members of insect societies resembles the system that members of vertebrate societies follow.
- Ⓔ Major structural differences exist between insect and vertebrate societies.

Why would most examinees answer this question incorrectly? The reason is simple: it is easy to confuse statements made about specific insect and vertebrate societies with statements made about insect and vertebrate societies in general. In this passage, in the fourth sentence, the author switches from talking about Wilson's views

of insect and vertebrate societies in general and refers to his comments on termites and macaques in specific.

Go through the answer choices one by one. Does the passage suggest that a clearly-defined division of labor distinguishes *most* insect and vertebrate societies? No. It merely states that, according to Wilson, a clearcut division of labor is a characteristic of termite and rhesus macaque societies. Choice A is incorrect: you cannot justify leaping from a single type of insect (*termites*) and a single type of vertebrate (*rhesus macaques*) to most insects and most vertebrates.

Does the passage suggest that the caste structure of insect societies shares certain likenesses with that of their counterparts in vertebrate societies? No. It merely states that, according to Wilson, termites and macaques both can communicate rank within a particular caste. Choice B is incorrect. You cannot assume that the caste structure of insect societies is similar to the caste structure of vertebrate societies just because termites and rhesus macaques both have some way to communicate caste status or rank.

Does the passage suggest that *most* insect and vertebrate societies form cooperative groups in order to hold and defend their home range or territory? No. It merely states that termites and macaques organize themselves into cooperative groups, and that both species occupy and defend territories. Choice C is incorrect: again, you cannot justify leaping from termites and rhesus macaques to *most* insects and *most* vertebrates.

Does the passage suggest that the system of communication employed by members of insect societies resembles that employed by members of vertebrate societies? No. It merely states that communication among termites and macaques serves similar ends; it says nothing about the specific systems of communication they use, nor about those systems of communication used by other insects and vertebrates. Choice D is incorrect.

The correct answer is Choice E. In the passage, the author states that Wilson has grown less impressed "with the structural differences that divide them (i.e., insect and vertebrate societies) to such an apparently irreconcilable degree." This suggests that, even though Wilson may be unimpressed with them, these differences exist and are *major*.

TACTIC 8 When Asked to Apply Ideas from the Passage to a New Situation, Put Yourself in the Author's Place

GRE application questions require you to do three things:

1. *Reason* — If X is true, then Y must also be true.
2. *Perceive Feelings* — If the author feels this way about subject A, he probably feels a certain way about subject B.
3. *Sense a Larger Structure* — This passage is part of an argument for a proposal, or part of a description of a process, or part of a critique of a hypothesis.

Like inference questions, application questions require you to go beyond what the author explicitly states. Application questions, however, ask you to go well beyond a simple inference, using clues in the passage to interpret possible reasons for actions and possible outcomes of events. Your concern is to comprehend how the author's ideas might apply to other situations, or be affected by them. To do so, you have to put yourself in the author's place.

Imagine you are the author. What are you arguing for? Given what you have just stated in the passage, what would you want to say next? What might hurt your argument? What might make it stronger? What kind of audience would appreciate what you have to say? Whom are you trying to convince? If you involve yourself personally with the passage, you will be better able to grasp it in its entirety and see its significance.

Answer the following application question based on the previous passage discussing Wilson's *Sociobiology*.

EXAMPLE

Which of the following statements would be most likely to begin the paragraph immediately following the passage?

- Ⓐ Wilson has raised a problem in ethical philosophy in order to characterize the essence of the discipline of sociobiology.
- Ⓑ It may not be too much to say that sociology and the other social sciences are the last branches of biology waiting to be integrated into neo-Darwinist evolutionary theory.
- Ⓒ Although behavioral biology is traditionally spoken of as if it were a unified subject, it is now emerging as two distinct disciplines centered on neurophysiology and sociobiology, respectively.
- Ⓓ The formulation of a theory of sociobiology constitutes, in Wilson's opinion, one of the great manageable problems of biology for the next twenty or thirty years.
- Ⓔ In the past, the development of sociobiology has been slowed by too close an identification with ethology and behavioral psychology.

As you know from answering the previous main idea and attitude questions, Wilson's point is that students of insect and vertebrate societies may be on the verge of devising a general theory of sociobiology. Like Wilson, the author of the passage appears optimistic about the likelihood of developing this unified science. At the same time, again like Wilson, he is cautious; he too does not wish to overstate the case.

Put yourself in the author's place. What would you be likely to say next? The author has just been describing Wilson's hopeful view of the prospects for putting together a general theory of sociobiology. What would be more natural than for him next to discuss Wilson's opinion of a time frame for formulating this general theory? Choice D, with its confident yet judicious view of the formulation of a theory of sociobiology as "one of the great *manageable* problems of biology for the next twenty or thirty years," seems a logical extension of what the passage has just been saying. While Choices A, B, C, and E all touch on sociobiology in some way, none of them follows as naturally from the passage's immediate argument.

TACTIC 9 When Asked to Give the Meaning of an Unfamiliar Word, Look for Nearby Context Clues

When a question in the reading comprehension part of an examination asks for the meaning of a word, that meaning can usually be deduced from the word's context. The purpose of this kind of question is to determine how well you can extract meaning from the text, not how extensive your general vocabulary is.

Sometimes the unknown word is a common word used in one of its special or technical meanings. For example:

> He *threw* the pot in an hour. The wheel turned busily and the shape grew quickly as his fingers worked the wet, spinning clay. (*Throw* here means to shape on a potter's wheel.)

At other times, the unknown word may bear a deceptive resemblance to a known word.

> He fell *senseless* to the ground. (He was unconscious. He did not fall foolishly or nonsensically to the ground.)

Just because you know *one* meaning of a word, do not assume that you know its meaning as it is used in a particular passage. You must look within the passage for clues. Often authors will use an unfamiliar word and then immediately define it within the same sentence. The two words or groups of words are juxtaposed — set beside one another — to make their relationship clear. Commas, hyphens, and parentheses may signal this relationship.

1. The *rebec*, a medieval stringed instrument played with a bow, has only three strings.
2. *Paleontologists* — students of fossil remains — explore the earth's history.
3. Most mammals are *quadrupeds* (four-footed animals).

Often an unfamiliar word in one clause of a sentence will be defined or clarified in the sentence's other clause.

1. The early morning dew had frozen, and everything was covered with a thin coat of *rime*.
2. Cowards, we use *euphemisms* when we cannot bear the truth, calling our dead "the dear departed," as if they have just left the room.

Refer once more to the passage on visual recognition to answer the following question.

> What is involved in the process of visual recognition? First, like computer data, visual memories of an object must be stored; then, a mechanism must exist for them to be retrieved. But how does this process work? The eye triggers the nerves into action. This neural activity constructs a picture in the brain's memory system, an internal image of the object observed. When the eye once again confronts that object, the object is compared with its internal image; if the two images match, recognition takes place.
>
> Among psychologists, the question as to whether visual recognition is a parallel, single-step operation or a sequential, step-by-step one is the subject of much debate. Gestalt psychologists contend that objects are perceived as wholes in a parallel operation: the internal image is matched with the retinal impression in one single step. Psychologists of other schools, however, suggest the opposite, maintaining that the individual features of an object are matched serially with the features of its internal image. Some experiments have demonstrated that the more well-known an object is, the more holistic its internal image becomes, and the more parallel the process of recognition tends to be. Nonetheless, the bulk of the evidence appears to uphold the serial hypothesis, at least for simple objects that are relatively unfamiliar to the viewer.

EXAMPLE

Which of the following phrases could best replace "the more holistic its internal image becomes" (line 17) without significantly changing the sentence's meaning?

Ⓐ the more its internal image increases in detail
Ⓑ the more integrated its internal image grows
Ⓒ the more its internal image decreases in size
Ⓓ the more it reflects its internal image
Ⓔ the more indistinct its internal image appears

What words or phrases in the vicinity of "the more holistic its internal image becomes" give you a clue to the phrase's meaning? The phrase immediately following, "becomes more parallel." If the recognition process becomes more parallel as an object becomes more familiar, then matching takes place in one step in which all the object's features are simultaneously transformed into a single internal representation. Thus, to say that an object's internal image becomes more holistic is to say that it becomes more *integrated* or whole. The correct answer is Choice B.

TACTIC 10 Familiarize Yourself with the Technical Terms Used to Describe a Passage's Organization

Another aspect of understanding the author's point is understanding how the author organizes what he has to say. You have to understand how the author makes his point, figure out whether he begins with his thesis or main idea or works up to it gradually. Often this means observing how the opening sentence or paragraph relates to the passage as a whole.

Here is a technique question based on the last two sentences of the passage about sociobiology. Those lines are repeated here so that you can easily refer to them.

> Wilson readily concedes that, from a specialist's perspective, such a likeness may at first appear superficial, even unscientifically glib. Nonetheless, in this eminent scholar's judgment, "it is out of such deliberate oversimplification that the beginnings of a general theory are made."

EXAMPLE

Which of the following statements best describes the organization of the author's discussion of the importance of the termite/macaque comparison in the development of a unified science of sociobiology (lines 16–20)?

(A) He provides an example of a comparison and then rejects its implications.
(B) He concedes that current data are insufficient and modifies his initial assertion of their importance.
(C) He acknowledges hypothetical objections to the comparison, but concludes by reaffirming its significance.
(D) He cites critical appraisals of the comparison, but refrains from making an appraisal of his own.
(E) He notes an ambiguity in the comparison, but finally concedes its validity.

Consider the first clause of each answer choice.

In his comment on how things may seem from the specialist's point of view, does the author *provide an example* of a comparison? No. He refers to a comparison made earlier. Therefore, you can eliminate Choice A.

Does he *concede the insufficiency* of current data? Not quite. He states that some people may quarrel with the comparison because it seems glib to them; he does not grant that they are right or that the data are inadequate. Therefore, you can eliminate Choice B.

Does he *acknowledge hypothetical objections* to the comparison? Definitely. Make a note to come back later to Choice C.

Does he *cite critical appraisals* of the comparison? Possibly. Again, make a note of Choice D.

Does he *note an ambiguity* in the comparison? No. He notes an objection to the comparison; he mentions no ambiguities within it. Therefore, you can eliminate Choice E.

Now consider the second clause of Choices C and D. Does the author *refrain from making an appraisal* of the comparison? No. He calls it a deliberate oversimplification that may bear fruit. Choice D is incorrect. Does the author conclude by *reaffirming the significance* of the termite/macaque comparison? Clearly he does; he quotes Wilson's conclusion that such oversimplified comparisons can provide the basis for an important general theory. The correct answer is Choice C.

TACTIC 11 In Answering Logical Reasoning Questions, Read Each Argument Very Carefully

Some students, who find that they can answer many reading comprehension questions correctly by skimming the passage without reading every word, attack logical reasoning questions in the same way. This is a very poor strategy.

First of all, the temptation to skim logical argument passages should be less, since these passages are much shorter than the usual run of reading comprehension passages, and skimming them will save less time. More important, in logical reasoning passages, it is not enough to have a general idea about the argument; you must be able to analyze the argument very closely.

A cursory reading is not sufficient to pick up a subtle flaw in logic or to ascertain what unstated premise the author is assuming to be true.

TACTIC 12 In Tackling Logical Reasoning Questions, Always Identify the Conclusion of the Argument

It is imperative that you are absolutely clear about what conclusion the author of the argument claims to have reached. The three most common situations are as follows:

- The conclusion is the last sentence of the passage, often introduced by a word such as *therefore*, *so*, *thus*, *hence*, or *consequently*. Here is a simple example of this type of argument:

  Joan Smith has those qualities that we seek in our congressional leaders. She is honest, hardworking, intelligent, and dedicated. Having served for ten years in the House of Representatives, she has the requisite experience to be an effective United States Senator. Therefore, you should enthusiastically vote for Ms. Smith in this year's election.

- The conclusion is the first sentence of the passage, followed by the supporting evidence. In such a case, there is no word such as therefore signaling the conclusion, but it is still very easy to spot. For example, the preceding argument could have been presented as follows:

  Joan Smith deserves your vote for United States Senator. She has those qualities that we seek in our congressional leaders. She is honest, hardworking, intelligent, and dedicated. In addition, having served for ten years in the House of Representatives, she has the requisite congressional experience to be an effective United States Senator.

- The conclusion is not in the passage. In such cases, the question usually asks you to identify the conclusion that is implicit in the argument. For example, if in the two preceding arguments the last or first sentence, respectively, had been omitted, you would have had no difficulty determining that the author of the passage wanted you to vote for Joan Smith. The question might have asked, "Which of the following five statements can most reasonably be inferred from the statements in the given passage?"

TACTIC 13 In Tackling Logical Reasoning Questions, Pay Particular Attention to Signal Words in the Question (and in the Argument As Well)

In answering logical reasoning questions, you must read closely both the argument and the question or questions based on it. When you do so, be on the lookout for certain signal words that can clarify the situation. In particular, be alert for:

Cause and Effect Signal Words

The following words often signal the conclusion of an argument:

accordingly	so
consequently	therefore
for this reason	thus
hence	

Contrast Signal Words

The following words often suggest a reversal of thought within an argument or question stem:

although	instead
but	nevertheless
despite	not
even though	on the contrary
except	on the other hand
however	rather than
in contrast	unlike

Notice that in the following logical reasoning problem several of these words are present: the argument contains the words *despite*, *not*, and *consequently*, and the question stem has the word *except*. Each of these words plays a role in your reasoning.

> Despite the fact that River City increased the average class size by more than 15% in all grades two years ago, this year's average SAT scores for the junior class were the highest ever. This shows that class size is not a good determinant of student performance. Consequently, other school districts should follow River City's lead and save money by increasing the size of their classes.

EXAMPLE

Each of the following statements, if true, is a valid objection to this argument EXCEPT:

(A) The advantages of smaller classes are more pronounced in elementary school than in high school.
(B) The number of classroom discipline problems reported by teachers is directly proportional to the number of students in the classroom.
(C) Japanese schools have a lower teacher-to-student ratio than American schools do and have generally better results on international standardized tests.
(D) Three years ago, the eighth graders in River City Middle School had very high scores on their standardized tests.
(E) The effects on students of learning in larger classes take at least three or four years to manifest themselves completely.

It is implicit in the question stem that the argument is not very persuasive, and that there are several possible objections to it that could be raised. In fact, the question stem tells you that four of the five statements listed raise valid objections to the argument presented. Your job is to determine the only one that does not.

The conclusion that larger class sizes are not detrimental to student learning is based on a single piece of data concerning high school juniors.

Choice A raises the objection that looking at the results of high school students on the SAT does not tell the whole story and that elementary school students will suffer from the larger classes.

Choice E raises an even stronger objection. It suggests that all students may suffer the consequences of increased class sizes; it will just take more time until the results are clearly discernible.

Choice B raises a completely different objection. Even if student academic performance is not adversely affected by larger class sizes, there are behavioral disadvantages to having large classes.

Choice D raises still another objection to the argument, the support for which is based on the performance of this year's junior class. Because three years ago, as eighth graders, the members of this class had very high test scores, it is possible that this group of students is brighter than the average. If so, it is likely that they would excel regardless of class size, whereas other students might suffer more.

Choice C is slightly harder to analyze. If the word *lower* makes you think *smaller*, Choice C seems to say that smaller classes, at least in Japan, result in higher test scores, and are thus beneficial. This then would be yet another valid objection to the given argument. If, however, you are confident in your analysis to this point and are sure that Choices A, B, D, and E are incorrect, by the process of elimination, Choice C *must* be the correct answer. So look at Choice C again. In fact, Choice C refers to a lower teacher-to-student ratio. A lower teacher-to-student ratio means more students per teacher, not fewer students. If there are more students per teacher, that means there will be larger class sizes, not smaller. Choice C then is not an objection to the argument; it supports the argument by showing that good results can occur in larger classes.

As this example shows, logical reasoning reading questions must be read very carefully. Do not attempt to analyze them too quickly.

TACTIC 14 Always Use the Process of Elimination to Reject Incorrect Choices

From Tactic 1, you know that in logical reasoning reading questions, as in all computer-based reading questions, you should always read the question first. This, of course, does not guarantee that you will know the correct answer before you read the answer choices; in fact, more often than not, you won't. What do you do then? Use the process of elimination. In the best-case scenario, using the process of elimination will allow you to zoom in on the correct answer; at worst, it will eliminate some obvious wrong choices and allow you to make an educated guess and move on.

See how the process of elimination works on the next logical reasoning reading question.

> In the United States between 1993 and 1998, the number of people on death row continued to increase, but at a rate lower than that of the general prison population.

EXAMPLE

Which of the following statements directly contradicts this claim?

Ⓐ The number of death row inmates increased slightly from 1993 to 1998.
Ⓑ Among people convicted of murder, the proportion of those who were sentenced to death decreased from 1993 to 1998.
Ⓒ Each year from 1993 to 1998, more death row inmates were executed than in the previous year.
Ⓓ Each year from 1993 to 1998, fewer people were sentenced to death than in the previous year.
Ⓔ The proportion of death row inmates among the general prison population rose from 0.6% in 1993 to 0.8% in 1998.

Even though the passage is only one sentence long, you should have read the question "Which of the following statements directly contradicts this claim?" first. Unfortunately, there are many ways to contradict the claim made in that sentence. So there is no point in trying to think of one, and then looking to see if it is one of the five choices. You simply must read each choice, and then, by process of elimination, find the correct one.

Caution

Do not spend even one second deciding whether you think the claim in the passage or any of the choices is true. This is completely irrelevant. Examine only the logic of the argument. Look for a statement that, if true, would mean that the claim is false.

- The passage states that the death row population increased. Choice A confirms this (and says nothing about the general prison population). Choice A is incorrect.
- Choice B compares the proportion of new death row inmates to the number of people convicted of murder, not to the general prison population. Choice B is incorrect.
- Choice C states that the number of people executed each year went up. If the number of people executed each year went up, the death row population might have decreased (thereby contradicting the first part of the claim), but not necessarily (not if they were replaced by many more people being sentenced to death). Choice C is incorrect.
- Choice D doesn't guarantee that the claim is true, but it comes closer to confirming it than to contradicting it. Even if fewer people were sentenced to death each year, some still were, so the number of people on death row might have increased. Again, this answer choice makes no reference to the general prison population. Choice D is incorrect.

Choice E is a little harder to analyze. Because it refers to an increase, many students would not choose it, thinking it confirms rather than refutes the claim. However, you must analyze it. Having definitively rejected choices A, B, C, and D, you know, by the process of elimination, that Choice E must be the correct answer. Let's examine why, in fact, it is.

- The passage claims the death row population increased at a slower rate than the prison population did. This means that the proportion of death row inmates in the prison population actually decreased. Choice E, which states that the proportion increased, is a direct contradiction of that claim.

NOTE

1. Although any of the choices A, B, C, and D could be true without the claim's being true, none of them is inconsistent with the truth of the claim.
2. Choices B, C, and D each introduce an extraneous issue. None of the following—the number of murder convictions, the number of executions, the number of people sentenced to death—is directly relevant to the claim.

TACTIC 15 In Questions About Weakening or Strengthening an Argument, Examine the Argument for Any Unstated Assumptions It Makes

An argument is based upon certain assumptions made by its author. If an argument's basic premises are sound, the argument is strengthened. If the argument's basic premises are flawed, the argument is weakened.

Pinpoint what the argument assumes. Then compare that assumption with the answer choices. If the question asks you to choose an answer that most strengthens the argument, look for the answer choice that is most in keeping with the argument's basic assumption. If the question asks you to choose an answer that most weakens the argument, look for the answer choice that casts the most doubt on that assumption.

Apply this tactic to the following question.

> In a recent speech, the president of a major college said, "It is extremely valuable for college-educated adults entering the workplace to be able to speak at least one foreign language fluently. I am, therefore, proposing that all of our students be encouraged to spend their junior year abroad."

EXAMPLE

Which of the following, if true, most weakens the president's argument?

Ⓐ Most students who study abroad for a full year return home with a good working knowledge of the language spoken in the country.
Ⓑ Only students who already know a language well will choose to study in a country where that language is spoken.
Ⓒ Some colleges do a much better job than others in teaching foreign languages.
Ⓓ Some students learn to speak foreign languages fluently by taking intensive immersion courses in the United States.
Ⓔ Many students who spend their junior year abroad learn to speak the language fluently, but cannot read and write with ease.

The argument claims that, in order for students to learn to speak foreign languages well, they should study abroad. It clearly assumes a high correlation between studying in a foreign country and learning to speak the language well. It assumes, at the least, that students who have studied abroad can speak a foreign language well, and, possibly, that students who have not studied abroad cannot.

Choice A is in keeping with the assumption inherent in the president's argument. If true, it would strengthen the argument, not weaken it. Choice E, by stating that many students who study abroad do not learn to read and write the language well, seems to cast doubt on the value of the junior year abroad program. However, since the president talked only about the value of being able to speak a foreign language well, Choice E also strengthens his argument.

Choices B and C are also incorrect. They neither strengthen nor weaken the president's argument. At worst, Choice B suggests that it may be difficult to convince some students to study abroad; however, it does not state that they should not be encouraged to do so. In order to weaken the president's argument, Choice C would have to go much further than it does; it would have to state explicitly that some colleges do such a good job that their students actually learn to speak foreign languages fluently.

The correct answer is Choice D. It states that it is possible for American students to learn to speak foreign languages fluently without studying abroad. Choice D weakens the president's argument. It does so by suggesting an alternative method by which college students could achieve the president's goal of speaking a foreign language fluently.

Practice Exercises

Note: Although the reading passages on the computer-based GRE range from 50 to 400 words in length, the paper-based GRE taken by students in foreign countries includes reading passages of up to 800 words in length. Therefore, the following practice exercises present a selection of long and short passages to help students to prepare for either the computer-based or the paper-based test.

Directions: **Each of the following reading comprehension questions is based on the content of the following passage. Read the passage and then determine the best answer choice for each question. Base your choice on what this passage states directly or implies, not on any information you may have gained elsewhere.**

One phase of the business cycle is the expansion phase. This phase is a twofold one, including recovery and prosperity. During
Line the recovery period there is ever-growing
(5) expansion of existing facilities, and new facilities for production are created. More businesses are created and older ones expanded. Improvements of various kinds are made. **There is an ever-increasing optimism about**
(10) **the future of economic growth**. Much capital is invested in machinery or "heavy" industry. More labor is employed. More materials are required. As one part of the economy develops, other parts are affected. For exam-
(15) ple, a great expansion in automobiles results in an expansion of the steel, glass, and rubber industries. Roads are required; thus the cement and machinery industries are stimulated. Demand for labor and materials results
(20) in greater prosperity for workers and suppliers of raw materials, including farmers. This increases purchasing power and the volume of goods bought and sold. Thus, prosperity is diffused among the various segments of the
(25) population. This prosperity period may continue to rise and rise without an apparent end. However, a time comes when this phase reaches a peak and stops spiraling upwards. This is the end of the expansion phase.

1. Which of the following statements best exemplifies the optimism mentioned in the boldfaced sentence of the passage as being part of the expansion phase?
   - Ⓐ Public funds are designated for the construction of new highways designed to stimulate tourism.
   - Ⓑ Industrial firms allocate monies for the purchase of machine tools.
   - Ⓒ The prices of agricultural commodities are increased at the producer level.
   - Ⓓ Full employment is achieved at all levels of the economy.
   - Ⓔ As technology advances, innovative businesses replace antiquated firms.

2. It can be inferred from the passage that the author believes that
   - Ⓐ when consumers lose their confidence in the market, a recession follows
   - Ⓑ cyclical ends to business expansion are normal
   - Ⓒ luxury goods such as jewelry are unaffected by industrial expansion
   - Ⓓ with sound economic policies, prosperity can become a fixed pattern
   - Ⓔ the creation of new products is essential for prosperity

3. Which of the following statements would be most likely to begin the paragraph immediately following the passage?
   - (A) Union demands may also have an effect on business cycles.
   - (B) Some industries are, by their very nature, cyclical, having regular phases of expansion and recession.
   - (C) Information is a factor that must be taken into consideration in any discussion of the expansion phase.
   - (D) The farmer's role during the expansion phase is of vital importance.
   - (E) The other phase of the business cycle is called the recession phase.

Both plants and animals of many sorts show remarkable changes in form, structure, growth habits, and even mode of reproduction in
Line becoming adapted to a different climatic envi-
(5) ronment, type of food supply, or mode of living. This divergence in response to evolution is commonly expressed by altering the form and function of some part or parts of the organism, the original identity of which is clearly dis-
(10) cernible. For example, the creeping foot of the snail is seen in related marine pteropods to be modified into a flapping organ useful for swimming, and is changed into prehensile arms that bear suctorial disks in the squids
(15) and other cephalopods. The limbs of various mammals are modified according to several different modes of life—for swift running (cursorial) as in the horse and antelope; for swinging in trees (arboreal) as in the monkeys;
(20) for digging (fossorial) as in the moles and gophers; for flying (volant) as in the bats; for swimming (aquatic) as in the seals, whales, and dolphins; and for other adaptations. The structures or organs that show main change in
(25) connection with this adaptive divergence are commonly identified readily as **homologous**, in spite of great alterations. Thus, the finger and wrist bones of a bat and whale, for instance, have virtually nothing in common
(30) except that they are definitely equivalent elements of the mammalian limb.

Directions: For the following question, consider each question separately and select *all* that apply.

4. The author provides information that would answer which of the following questions?
   - [A] What factors cause change in organisms?
   - [B] What is the theory of evolution?
   - [C] How are horses' legs related to seals' flippers?

5. Which of the following words could best be substituted for the boldfaced word **homologous** without substantially changing the author's meaning?
   - (A) altered
   - (B) mammalian
   - (C) corresponding
   - (D) divergent
   - (E) tactile

Although there are no physical differences between the visual organs of the two groups, the inhabitants of the Bilge Islands, when shown a card displaying a spectrum of colors, perceived fewer colors than do most persons in the United States.

6. Which of the following conclusions can most reliably be drawn from the information above?
   - (A) Human color perception is at least partly determined by factors other than the physical structure of the visual organs.
   - (B) The Bilge Islanders are probably taught in childhood to recognize fewer colors than are persons in the United States.
   - (C) Differences in social structure probably affect color perception.
   - (D) Color perception in humans is influenced by differences in physical environment.
   - (E) Bilge Islanders may have fewer terms denoting colors in their language than do English-speaking persons.

The layer of air next to the earth, which extends upward for about 10 miles, is known as the troposphere. On the whole, the tropo-
Line sphere makes up about 75% of all the weight
(5) of the atmosphere. It is the warmest part of the atmosphere because most of the solar radiation is absorbed by the earth's surface, which warms the air immediately surrounding it. A steady decrease of temperature with increasing
(10) elevation is a most striking characteristic of this region, whose upper layers are colder because of their greater distance from the earth's surface and because of the rapid radiation of heat into space. (Temperatures within
(15) the troposphere decrease about 3.5° per 1,000-foot increase in altitude.) Within the troposphere, winds and air currents distribute heat and moisture. Strong winds, called jet streams, are located at the upper levels of the
(20) troposphere. These jet streams are both complex and widespread in occurrence. They normally show a wave-shaped pattern and move from west to east at velocities of 150 mph, but velocities as high as 400 mph have been
(25) noted. The influences of changing locations and strengths of jet streams upon weather conditions and patterns are no doubt considerable. Current intensive research may eventually reveal their true significance.

7. It can be inferred from the passage that a jet plane will usually have its best average rate of speed on its run from
   - (A) New York to San Francisco
   - (B) Los Angeles to New York
   - (C) Boston to Miami
   - (D) Bermuda to New York
   - (E) London to Washington, DC

8. It can be inferred from the passage that at the top of Jungfrau, which is 12,000 feet above the town of Interlaken in Switzerland, the temperature is usually
   - (A) below freezing
   - (B) about 42° colder than on the ground
   - (C) warmer than in Interlaken
   - (D) affected by the ionosphere
   - (E) about 75° colder than in Interlaken

9. The passage states that the troposphere is the warmest part of the atmosphere because it
   - (A) is closest to the sun
   - (B) contains electrically charged particles
   - (C) radiates heat into space
   - (D) has winds and air currents that distribute the heat
   - (E) is warmed by the earth's heat

"The emancipation of women," James Joyce told one of his friends, "has caused the greatest revolution in our time in the most important
Line relationship there is—that between men and
(5) women." Other modernists agreed: Virginia Woolf, claiming that in about 1910, "human character changed," and, illustrating the new balance between the sexes, urged "Read the 'Agamemnon,' and see whether your
(10) sympathies are not almost entirely with Clytemnestra." D. H. Lawrence wrote, "perhaps the deepest fight for 2000 years and more, has been the fight for women's independence."

But if modernist writers considered women's
(15) revolt against men's domination one of their "greatest" and "deepest" themes, only recently—in perhaps the past 15 years—has literary criticism begun to catch up with it. Not that the images of sexual antagonism that
(20) abound in modern literature have gone unremarked; far from it. But what we are able to see in literary works depends on the perspectives we bring to them, and now that women—enough to make a difference—are reforming
(25) canons and interpreting literature, the landscapes of literary history and the features of individual books have begun to change.

10. According to the passage, women are changing literary criticism by
    - (A) noting instances of hostility between men and women
    - (B) seeing the literature from fresh points of view
    - (C) studying the works of early 20th-century writers
    - (D) reviewing books written by feminists
    - (E) resisting masculine influence

11. The author quotes James Joyce, Virginia Woolf, and D. H. Lawrence primarily in order to show that
    - (A) these were feminist writers
    - (B) although well-intentioned, they were ineffectual
    - (C) before the 20th century there was little interest in women's literature
    - (D) modern literature is dependent on the women's movement
    - (E) the interest in feminist issues is not new

When you first saw a piece of African art, it impressed you as a unit; you did not see it as a collection of shapes or forms. This, of course, means that the shapes and volumes within the sculpture itself were coordinated so successfully that the viewer was affected emotionally.

It is entirely valid to ask how, from a purely artistic point of view, this unity was achieved. And we must also inquire whether there is a recurrent pattern or rules or a plastic language and vocabulary that is responsible for the powerful communication of emotion which the best African sculpture achieves. If there is such a pattern of rules, are these rules applied consciously or instinctively to obtain so many works of such high artistic quality?

It is obvious from the study of art history that an intense and unified emotional experience, such as the Christian credo of the Byzantine or 12th or 13th century Europe, when espoused in art forms, gave great unity, coherence, and power to art. But such an integrated feeling was only the inspirational element for the artist, only the starting point of the creative act. The expression of this emotion and its realization in the work could be done only with discipline and thorough knowledge of the craft. And the African sculptor was a highly trained workman. He started his apprenticeship with a master when a child, and he learned the tribal styles and the use of tools and the nature of woods so thoroughly that his carving became what Boas calls "motor action." He carved automatically and instinctively.

12. The information in the passage suggests that a mature African carver might best be compared to a
    - (A) chef following a recipe
    - (B) fluent speaker of English just now beginning to study French
    - (C) batter who hits a home run the first time at bat
    - (D) veteran fiddler expertly varying a traditional tune
    - (E) senior editor correcting the prose of an unidiomatic author

The likelihood of America's exhausting her natural resources is growing less. All kinds of waste are being recycled, and new uses are constantly being found for almost everything. We are getting more use out of what we produce, and are manufacturing many new byproducts out of what we formerly threw away. It is, therefore, unnecessary for us to continue to ban logging in national parks, nature reserves, or areas inhabited by endangered species of animals.

13. Which one of the following most seriously undermines the conclusion of this argument?
    - (A) The increasing amount of recycled material made available each year is equal to one-tenth of the increasing amount of natural material consumed annually.
    - (B) Recent studies have shown that the number of endangered animals throughout the world fluctuates sharply and is chiefly determined by changes in meteorological conditions.
    - (C) The logging industry contributes huge sums of money to political campaigns in states where it has a financial interest.
    - (D) The techniques that make recycling possible are constantly improved so that more is reclaimed for lower costs each year.
    - (E) Political contributions by the recycling industry are now greater than those of either the logging or animal protection interests.

ANSWER KEY

1. **B**	6. **A**	11. **E**
2. **B**	7. **B**	12. **D**
3. **E**	8. **B**	13. **A**
4. **A, C**	9. **E**	
5. **C**	10. **B**	

PART 3

ANALYTICAL WRITING: TACTICS, STRATEGIES, AND PRACTICE

Introduction to Part 3

What sort of test is this new analytical writing test? First and foremost, it is not a multiple-choice test. It is a performance test—you have to write two analytical essays in one hour.

The new analytical writing section of the GRE is the most substantive of the three sections on the tests. This section is organized in two parts. In Part 1, "Present Your Perspective on an Issue," you have 30 minutes to write an essay expressing your point of view on a particular issue. You will be given a quotation that states an opinion about an issue; you will probably write a better essay if the quotation "grabs" you, but you can write a strong paper even if the topic seems unappealing at first.

Your job is to take a stand and to support it, drawing on your own experiences and on your readings to come up with examples that reinforce your argument. It does not matter what stand you take; there is no "correct" position, no one true answer. Many different approaches can work. You can agree completely with the quotation's point of view or you can dispute it absolutely. You can disagree with some aspects of the quote, but agree with others. What matters is how you present your case.

Part 2 of the analytical writing section asks you to perform a different but complementary task. In Part 2, "Analyze an Argument," you have 30 minutes to write an essay critiquing the logical soundness of an argument. You will be given one short passage in which an author makes a claim and backs it up, giving reasons that may well be flawed. You get no choice of passages to analyze; you must work with whatever passage comes up on your screen.

This time your job is not to advocate a particular point of view. This is not the moment for you to agree or disagree with the author; it is the moment for you to weigh the validity of the author's reasoning. Your approach is analytical and expository, not argumentative or persuasive. It is your task to examine carefully what the author offers as evidence. You will find it helpful to note what the author claims explicitly, and also to note what she or he assumes (not necessarily justifiably!).

If you study the tactics and work through the practice exercises in the following chapter, and take full advantage of the study materials on the GRE's website, *www.gre.org*, you will be well prepared for the analytical writing section of the GRE and should feel confident in your ability to write high-scoring essays.

CHAPTER 6

Analytical Writing

SCORING GUIDELINES

Two readers will judge your GRE analytical essays, awarding each essay a grade ranging from 0 to 6, with 6 the highest possible score. The powers-that-be then calculate your analytical writing score by taking the average of your four grades, rounding up the result to the nearest half-point. If one reader awarded your issues essay a 5 and your argument essay a 4, while the other reader gave both your essays 4's, you'd come out with a score of 4.25, rounded up to 4.5.

You probably have a sense of what score you need to be accepted by the graduate school of your choice. If you're seeking admission to Harvard's Ph.D. program in history, you're clearly aiming for a 5.5 or 6. If you're aiming for a graduate program in a field that favors number-crunching over essay-writing—mathematics or electrical engineering, for instance—you clearly don't need to aim so high. But however high a score you're seeking, you want to come out of the essay-writing section looking good. And to do that, you have to know what the GRE readers are looking for.

What are the GRE readers looking for? In essence, fluency, organization, and a command of technical English. These are the skills they assess.

Fluency

Fluency is smoothness and ease in communicating. In this case, it is your ability to set down a given number of words on paper within a limited period of time. If you freeze on essay examinations, writing only a sentence or two when whole paragraphs are called for, then you need to practice letting your words and ideas *flow*.

Literary fluency, however, involves more than just the number of words you type. The readers tend to award their highest grades to test-takers who use language well, those who employ a variety of sentence types and demonstrate a command of vocabulary. If you invariably use short, simple sentences, you need to practice constructing more complex ones. If you have a limited vocabulary, you need to expand it, working with *Barron's GRE Flash Cards* and other tools to learn the precise meaning of each new word you employ.

Organization

Organization is coherent arrangement. In this case, it is your ability to arrange your

basic unit of composition; the beginning of each new paragraph serves to alert readers that they are coming to a new step in the development of the subject. One paragraph leads to the next, drawing readers on to the essay's conclusion.

Organization involves your ability to reason and to marshal evidence to support your viewpoint. If you jump from subject to subject within a single paragraph, if you leave out critical elements, if you misorder your points or never manage to state exactly what you mean, then you need to practice outlining your position briefly before you express it in essay form.

Technical English

Technical English is the part of English that most students hate—grammar, spelling, punctuation, word usage. In this case, it is your ability to produce grammatically correct sentences in standard written English. If your English compositions used to come back to you with the abbreviations "frag" or "agr" or "sp" scribbled all over the margins, then you need to practice reading through your papers to catch any technical mistakes.

There are literally hundreds of handbooks available that will help you handle the mechanics of writing essays. Strunk and White's manual, *The Elements of Style,* provides clear, concise advice, as does William Zinsser's *On Writing Well.* Other good reference tools are *The Harbrace College Handbook,* Edward Johnson's *Handbook of Good English,* and, for the complete grammarphobe, Patricia O'Conner's aptly named *Woe Is I.*

NOTE: Unless you are someone who can't type two words in a row without making a spelling error, do not worry about spelling and punctuation mistakes. The GRE readers generally ignore them. However, if you make so many errors that it becomes difficult for the readers to make sense of what you have written, they will lower your score accordingly.

ESSAY-WRITING: THE 5-STEP APPROACH

How to Handle the Issue-Writing Task

You have 30 minutes to complete the issue-writing task. To earn a top score, you need to produce a smooth, 400–700-word essay with solid content, coherent organization, and few, if any, mechanical errors.

Each issue topic is presented as a 1–2 sentence statement commenting on a subject of general concern. This statement makes a claim. Your essay may support, refute, or qualify the views expressed in the statement. Whatever you write, however, must be relevant to the issue under discussion, and you must support your viewpoint with data—reasons and examples derived from your studies, experience, and reading.

GRE readers will evaluate your essay, grading it on the basis of your effectiveness in the following areas:

- Analysis of the statement's implications
- Organization and articulation of your ideas
- Use of relevant examples and arguments to support your case
- Handling of the mechanics of standard written English

Here is a 5-step plan you can use in writing your issue essay. Suggested times are approximate.

30 Minutes to Write an Issue Essay

- 2 Minutes **Brainstorm**
- 3 Minutes **Outline**
- 15 Minutes **Speed-write**
- 5 Minutes **Open and Close**
- 5 Minutes **Reread and Revise**

Keep one eye on the clock!

Step One: Begin with Brainstorming (2 minutes)

You do not lack ideas. What you may lack is a direct means of getting in touch with the ideas you already have. One useful technique to "prime the pump" and encourage fluency is *clustering.* Clustering is a method of brainstorming in which you start with a key word or short phrase and let that word or phrase act as a stimulus, triggering all sorts of associations that you jot down. In just a minute or two, you can come up with dozens of associations, some of which you may later be able to incorporate into your essay. (For a thought-provoking discussion of clustering and other brainstorming techniques, see *Writing the Natural Way* by Gabriele Rico.)

Let the issue statement or prompt trigger your brainstorming. As soon as you've clicked on your chosen topic, grab your pencil and sum up the claim the author is making. If, for example, the issue prompt is "Historians and other social scientists are as useful to society as are biochemists and engineers because society's ills cannot be cured by technological progress alone," your quick summation might be "Historians are as useful as scientists." Once you're clear about the author's point, start scribbling. Write down as many reasons that support or weaken the author's claim as you possibly can. Be sure to write both reasons *for* and reasons *against.* Don't worry right now if any of these reasons strike you as flimsy or implausible or clichéd; you can always cut them later or find ways to strengthen them, if you need to. Just note them down on your scratch paper, together with examples supporting both sides of the issue. Stay loose; this is your time for free associations, not self-censorship.

Step Two: Organize Your Outline (3 minutes)

According to British rhetorical theorist and philosopher Stephen Toulmin, a sound argument requires three elements: CLAIM, GROUNDS (or data), and WARRANT. Your claim is your thesis; it is an overall statement of the argument you hope to prove. The grounds for your argument are your evidence. Grounds for an argument can include statistics, examples, and even anecdotes. The warrant is the connection between the claim and the grounds. It is an explanation of how the grounds justify the claim.

CLAIM (thesis): Historians and other social scientists are as useful to society as are biochemists and engineers because society's ills cannot be cured by technological progress alone.

Once you have settled on your claim, look to your brainstorming for the arguments that support it. Each of these arguments requires its own claim, grounds, and warrant.

1. CLAIM: War is not prevented by technological progress.
   GROUNDS: Invention of gunpowder, nuclear weapons.
   WARRANT: Technological progress is driven by war; in fact, technology tends to make war more destructive.
2. CLAIM: Historians and social scientists can prevent, or at least discourage, war through their understanding of why wars have occurred in the past.
   GROUNDS: Treaty of Versailles, Marshall Plan.

WARRANT: An understanding of history can allow us to design policies that encourage peace.

3. CLAIM: Technological progress does not prevent poverty.
   GROUNDS: Industrial Revolution, sweatshops.
   WARRANT: Technology changes the distribution of wealth, increasing extreme poverty as it increases wealth for some.
4. CLAIM: Historians and social scientists can prevent poverty through economic policy.
   GROUNDS: New Deal, Social Security.
   WARRANT: Social programs prevent poverty.

Though not a necessary component of the argument, RESERVATIONS can strengthen a claim. A reservation is a rebuttal to the claim that is introduced and granted by the writer. Reservations strengthen arguments in several ways: First, they moderate the writer's claim, thereby decreasing the level of proof required. Second, reservations make the writer appear more reliable by demonstrating that she is open-minded, and that her position is not extreme. Third, reservations allow the writer to defuse criticism before it is made. When you include a reservation in your argument, be sure to take the opportunity to weigh it against your other claims.

5. RESERVATION: Biochemists and engineers do contribute to society.
   WEIGHING: Though technological progress can increase the food supply and cure disease, we will always need historians and social scientists to show us how to use technology without causing more harm than good.

Step Three: Write the Body of Your Essay (15 minutes)

You already know your general line of reasoning, the direction you want your argument to take. You need to spend the bulk of your time writing the body of your essay. As rapidly as you can, type up your points, writing two to three sentences to flesh out each reason or example in your outline. Do not worry if time pressure doesn't allow you to deal with every point you dreamed up. Start with a reason or example that you can easily put into words, preferably your best, most compelling reason or example. Given the 30-minute time limit you're working under, you want to be sure to cover your best points right away, before you run out of time. During the revision period, you can always rearrange your paragraphs, putting the strongest paragraph immediately before the conclusion, so that your essay builds to a solid climax.

Step Four: Now Write Your Opening and Summary Paragraphs (5 minutes)

It may seem strange to write your introductory paragraph after you have written the body of your essay, but it is a useful technique. Many writers launch into writing the introduction, only to find, once they have finished the essay, that their conclusion is unrelated to, or even contradicts, what they had written in the introduction. By writing the introduction *after* you have composed the bulk of the essay, you will avoid having to rewrite the introduction to support the conclusion that you *actually* reached, rather than the conclusion that you *expected* to reach.

This is one area in which the technology of the new GRE will greatly assist you. If the GRE were a hard copy (paper) exam, you would need to save space on your page to insert your introduction, guessing exactly how much room you would need. Instead, because the GRE is computerized, you can simply go back to the top of the page and begin writing the introduction.

What then should your introduction include? Your introductory paragraph should both introduce the topic on which you are writing and clearly indicate your thesis or point. While in some situations it is strategic (or simply more graceful) to reveal your thesis fully only in the conclusion, the GRE is *not* one of those situations. Clarity is key; you do not want to risk leaving your readers uncertain of your line of reasoning, or under the impression that you have strayed from the point.

For a top score, your introductory paragraph should also provide some context for the argument. The GRE readers appear to favor introductions that place the topic in an historical or social context, rather than simply discussing it in a contextual vacuum. The two introductory paragraphs below demonstrate the difference between these two types of introduction.

Introduction with Context

> Western society tends to glorify the individual over the group. Our social and political philosophy, based on John Stuart Mill's faith that progress is fostered by competition within the marketplace of ideas, encourages people, as the Apple computer commercial says, to "think different." This cult of the individual overemphasizes the importance of being different and fails to recognize that a healthy person will be both a conformist and an individualist. Ironically, self-conscious dedication to nonconformity will ultimately result in extreme slavishness to custom.

Introduction without Context

> A healthy individual is neither a conformist, nor an individualist; he is *both* a conformist and an individualist. Balancing conformity and individualism allows people to follow their interests and passions without wasting time on issues that do not interest them, while a self-conscious dedication to nonconformity ultimately results in an extreme slavishness to custom.

One last note on introductions: While you may have been taught in school that a paragraph must comprise at least three sentences, the GRE readers are not concerned about the length of your introductory paragraph. In fact, they appear willing to grant the highest score to essays whose introduction is only one sentence long. This does not mean that they favor essays with single-sentence introductions, only that they do not discriminate against them. If your introduction makes your thesis clear, it has done its job.

Your conclusion should, however, be longer than one sentence. It should restate your thesis and summarize the arguments that you make in its support. You should mention your supporting arguments in the same order in which they appear in the body of the essay. This technique underscores the organization of your essay, giving it a predictable and orderly appearance.

Step Five: Reread and Revise (5 minutes)

Expert writers often test their work by reading it aloud. In the exam room, you cannot read out loud. However, when you read your essay silently, take your time and listen with your inner ear to how it sounds. Read to get a sense of your essay's logic and rhythm. Does one sentence flow smoothly into the next? Would they flow more smoothly if you were to add a transition word or phrase (*therefore, however, nevertheless, in contrast, similarly*)? Do the sentences follow a logical order? Is any key idea or example missing? Does any sentence seem out of place? How would things sound if you cut out that awkward sentence or inserted that transition word?

Take a minute to act on your response to hearing your essay. If it sounded to you as if a transition word was needed, insert it. If it sounded as if a sentence should be cut, delete it. If it sounded as if a sentence was out of place, move it. Trust your inner ear, but do not attempt to do too much. Have faith in your basic outline for the essay. You have neither the need nor the time to revise everything.

Now think of yourself as an editor, not an auditor. Just as you need to have an ear for problems of logic and language, you also need to have an eye for errors that damage your text. Take a minute to look over your essay for problems in spelling and grammar. From your English classes you should know which words and grammatical constructions have given you trouble in the past. See whether you can spot any of these words or constructions in your essay. Correct any really glaring errors that you find. Do not worry if you fail to catch every mechanical error or awkward phrase. The readers understand that 30 minutes doesn't give you enough time to produce polished, gemlike prose. They won't penalize you for an occasional mechanical glitch.

HOW TO HANDLE THE ARGUMENT-ANALYSIS TASK

You have 30 minutes to complete the argument-analysis task. To earn a top score, you need to produce a smooth, 300–400 word critique with solid content, coherent organization, and few, if any, mechanical errors.

As you critique the argument, think about the writer's underlying assumptions. Ask yourself whether any of them are questionable. Also evaluate any data or evidence the writer brings up. Ask yourself whether this evidence actually supports the writer's conclusion.

In your analysis, you may suggest additional kinds of evidence to reinforce the writer's argument. You may also suggest methods to refute the argument, or additional data that might be useful to you as you assess the soundness of the argument. *You may **not**, however, present your personal views on the topic.* Your job is to analyze the elements of an argument, not to support or contradict that argument.

GRE readers will evaluate your essay, grading it on the basis of your effectiveness in the following areas:

- Identification and assessment of the argument's main elements
- Organization and articulation of your thoughts
- Use of relevant examples and arguments to support your analysis
- Handling of the mechanics of standard written English

Again, follow a 5-step approach in dealing with the argument-analysis task.

Step One: Identify the Claims (2 minutes)

Before you can identify the flaws in an argument essay prompt, you must have a clear understanding of the claims it makes. After reading the prompt once for general understanding, examine it more carefully, one sentence at a time. As you do this, use your scratch paper to write a list of the claims made in the prompt. List the claims in the order in which they are made. GRE argument prompts typically contain at least three flaws in the author's reasoning or use of evidence.

Here is an example of the notes you might take if you were writing on the topic below.

Discuss how effective you find the reasoning in this argument.

The following appeared in an article in the Real Estate section of the Springfield Bugle.

> Springfield is a great place to live. Every year, hundreds of former city dwellers move to Springfield, spurning the sophisticated cultural offerings of the urban setting for Springfield's more relaxed atmosphere. Despite the attractions of big city life, Springfield's new citizens choose their home for its rural setting and small town atmosphere. If Springfield wants to continue to attract these newcomers, it must adopt aggressive planning regulations to keep out chain stores, fast food establishments, bars, and other businesses more appropriate to an urban setting.

Overall Point: Springfield must control the growth of certain types of businesses in order for it to remain attractive to newcomers.
Claim One: People come to Springfield to get away from sophisticated city culture, and to have a relaxed atmosphere.
Claim Two: People come to Springfield for its rural, small-town atmosphere.
Claim Three: Keeping chain stores, bars, and fast food restaurants out of Springfield will maintain its attractiveness to newcomers.

Step Two: Question the Claims (3 minutes)

Once you have identified the claims made in the prompt, you need to assess the strength of those claims. In most cases, their shortcomings will be apparent to you. If, however, you are having trouble figuring out the flaws in a given claim, try applying a few handy questions to it.

1. GROUNDS. Is there any **evidence** to support the claim?
   The first two claims in the prompt above are assertions. Though the author might have survey data to support her claim that newcomers move to Springfield to escape urban culture and enjoy a more relaxed, rural, small-town atmosphere, she presents no such data in her argument.
2. WARRANT. Does the evidence provided support the claim?
   Could **other factors** cause the effect about which the author is writing? In the situation described in the prompt above, there are many possible reasons to

30 Minutes to Write an Argument Essay

- 2 Minutes **Spot the Claims**
- 3 Minutes **Question the Claims**
- 15 Minutes **Speed-write**
- 5 Minutes **Open and Close**
- 5 Minutes **Reread and Revise**

Keep one eye on the clock!

choose to move to Springfield. The author gives no reason for readers to believe that she has correctly identified the cause of Springfield's popularity.

Does the author assert a general rule based on an overly **small sample**? For example, if the author of the Springfield argument based her claims about why newcomers generally move to Springfield on the comments of a single new neighbor, her claims would lack adequate support. They would be unwarranted.

Does the author compare **comparable groups**? If, for example, the author of the Springfield argument attempted to support her claims about why newcomers move to Springfield with surveys of residents who moved to Springfield twenty years ago, she would have no basis to make claims about people who have moved to Springfield more recently.

Step Three: Write the Body of Your Critique, Following the Order of the Claims Made in the Prompt (15 minutes)

Organization is an important part of writing a clear and coherent essay. The simplest and best approach is to discuss the claims made in the prompt in the order in which they are presented. There is no reason to try anything tricky or fancy. The test-makers have given you an order. Use it. Using the structure of the prompt will save you time. It will also discourage you from writing a discursive essay that wanders unpredictably from one idea to another. High scores go to test-takers who write clear and well-reasoned essays. Creativity in this context is more likely to confuse your readers than to earn you extra points.

As we recommended in the previous section on the issue essay, spend the bulk of your time writing the body of your critique. Get those ideas onto the screen, allotting two to three sentences to each claim to flesh it out.

Step Four: Then Add Your Introductory and Summary Paragraphs (5 minutes)

While following the structure of the prompt is a handy way to organize the body of your critique, you still need to write an introduction and conclusion to your essay. Your introductory paragraph should provide a general overview of the criticisms you have made in the body of your essay. Do not give too much detail in the introduction; it is where you introduce, rather than explain, your analysis. *Present your points in the introduction in the same order in which they appear in the body of the essay.* By doing so, you will give your reader a clear idea of where you are going and what you intend to demonstrate. *In your conclusion, briefly restate the main points you have made in the body of your critique, and suggest one or two ways the author could have made his or her argument more persuasive.*

Step Five: Reread and Revise (5 minutes)

Once again, our recommendation is: First listen, then look. Begin by reading your essay silently, listening with your inner ear to how it sounds. Ask yourself whether one sentence flows smoothly into the next, and whether any transition words might help the flow. Consider whether any key idea or example might be missing or any sentence seems out of place. Do not make any major changes. Just tweak things slightly to improve your essay's sound and sense.

Now cast an eye over your essay, looking for mechanical errors. You know the sorts of grammatical constructions and spelling words that create problems for you. See whether you can spot any of them in your essay. Correct any errors that jump out at you.

Here is an example of an argument critique that follows the organization of the prompt:

Discuss how effective you find the reasoning in this argument.

The following appeared in an article in the Real Estate section of the Springfield Bugle.

> Springfield is a great place to live. Every year, hundreds of former city dwellers move to Springfield, spurning the sophisticated cultural offerings of the urban setting for Springfield's more relaxed atmosphere. Despite the attractions of big city life, Springfield's new citizens choose their home for its rural setting and small town atmosphere. If Springfield wants to continue to attract these newcomers, it must adopt aggressive planning regulations to keep out chain stores, fast food establishments, bars, and other businesses more appropriate to an urban setting.

Response to the Argument

Springfield may well be a great place to live, but the author of this article makes a number of unsubstantiated assumptions about the attributes that make Springfield an attractive home. Based on these assumptions, the author makes a bold proposal regarding zoning and city planning. Though this proposal is intended to maintain the positive attributes that bring new residents to Springfield, it may fail to achieve this goal or even have the perverse effect of worsening the quality of life in the town.

The author's first mistake is to assume that she knows why hundreds of former city dwellers move to Springfield each year. She claims that in moving to Springfield, people are rejecting the culture of the city in favor of Springfield's more relaxed suburban lifestyle. This is a classic case of confusing correlation with causation. While Springfield may in fact be more relaxed than the city, and while the city may have more sophisticated culture than Springfield, it does not follow that those who move from the city to Springfield are choosing relaxation over sophistication. Perhaps they are moving to Springfield for entirely different reasons. High urban property values, with their concomitant high urban property taxes, may be driving potential homeowners to less expensive suburban areas. People may also be moving to Springfield for better schools or a lower crime rate.

The claim that people move to Springfield for its small-town atmosphere and rural setting is similarly unsubstantiated. Yes, Springfield is a small, rural suburb. It does not follow, however, that this is why new residents move to Springfield. They could be moving to Springfield for any of the reasons mentioned above, or for any number of other reasons.

The conclusion that Springfield must keep out businesses that are common in urban areas if it is to remain an attractive community is unsupported. If new residents are really being drawn to Springfield by something other than the ways in which it is different from a big city, there is no reason to believe that keeping Springfield from growing city-like will make it more attractive. In fact, if people

move to Springfield in spite of its lack of big-city amenities and because of its lower cost (or some other factor), the addition of big-city businesses may make Springfield more attractive to newcomers.

Ironically, if the author is correct that Springfield's relaxed, small-town feel is what attracts new residents, making Springfield attractive to former city dwellers may, in the long run, destroy Springfield's positive attributes. After all, for how long can Springfield maintain this small-town atmosphere, if hundreds of newcomers are encouraged to move there each year? Ultimately, the author of this article appears to seek the impossible—a quiet small town with sustained, robust population growth.

Despite the flaws in this author's argument, she may be correct in her assessment of why newcomers move to Springfield. She could strengthen her argument by documenting its most important premise with data. If, for example, she provided survey results from newcomers, indicating that they did indeed come to Springfield to escape urban culture and to enjoy a more relaxed, rural, small-town atmosphere, her argument would be far more persuasive. Were this the case, her call for more restrictive zoning might be justified.

Testing Tactics

PREPARING FOR THE WRITING TEST

TACTIC 1 Take Advantage of the GRE's Free Study Aids

TIP

Download *PowerPrep II*—It's *Free!*

When you sign up to take the GRE General Test, you will eventually be sent *PowerPrep II*, a CD-ROM containing test preparation software for the General Test and Writing Assessment. However, you do not have to wait for your copy of *PowerPrep II* to come in the mail. You can download it immediately from the GRE website, *www.gre.org*.

PowerPrep II is helpful because it uses the same GRE word processing software that you will have to use to write your essays when you take your computer-based test. It is a very basic word processor that lets you perform very basic tasks. You can insert text, delete text, and move text around using a cut-and-paste function. You can also undo an action you've just performed.

Familiarize yourself with this word processing software so that, on the test date, you'll be comfortable using it. This software simulates actual testing conditions and presents actual essay topics. Practice writing your essays while you keep one eye on the clock. You need to develop a sense of how much time to allow for thinking over your essay and how much time to set aside for the actual writing.

A WORD OF WARNING

Attention, Mac users: *PowerPrep II* is compatible only with IBMs or PCs. If you own an Apple MacIntosh computer, you'll have to gain access to a PC to run *PowerPrep II*. Do it, even if it means making an extra trip to the campus computer lab or the nearest public library.

TACTIC 2 Practice Taking Shortcuts to Maximize Your Typing Efficiency

Slow and steady is not the way to go, at least not when you're taking the analytical writing test on the GRE. Fast typists have a decided advantage here. Unfortunately, you cannot turn yourself into a typing whiz overnight. However, you can use your time right now to practice some shortcuts to help you on the day of the test.

First, using the GRE's own word processing program (which comes when you download *PowerPrep II*), you can practice using the cut-and-paste function to copy phrases that you want to repeat in your essay. In an argument essay, for example, you might want to reuse such phrases as "the author makes the following assumption" or "another flaw in the author's argument is that...." In an issues essay, if you are running out of time and still haven't written your opening and summary paragraphs (which we advise you to compose *after* you've written the body of your text), you can write just your concluding paragraph, cutting and pasting it to both the beginning and end of the essay. Then, in a few seconds, you can change the wording of that initial paragraph so that it works as an introduction, not as a conclusion. How does that cliché about essay-writing go? "Tell them what you're going to tell them, tell them it, then tell them what you've told them." It's easy to do so, using cut-and-paste.

One thing to note: The GRE word processor currently lacks a copy function. To copy a chunk of text, you must first cut it and then directly paste it back in its original spot; next, you must move the cursor to the place where you want to reproduce the text and paste it there. The process may feel cumbersome at first, but by practicing with the word processor you will quickly build up speed copying using cut-and-paste.

Second, you can also practice abbreviating multiword names or titles. Consider the following argument topic or prompt:

Discuss how effective you find the reasoning in this argument.

The parent of a Collegiate High student included these remarks in a letter to the education page of the Oakville Bugle.

> If you look closely at Oakville's two leading private high schools—Collegiate Preparatory High School and Exover Academy—you must conclude that Collegiate is unmistakably superior to the Academy. Collegiate has a staff of 35 teachers, many of them with doctorates. In contrast, Exover has a staff of 22, several holding only a bachelor's degree. Moreover, Collegiate's average class size is 12, compared to Exover's average class size of 20; Collegiate's students receive much more individual attention than their peers do at the Academy. Students graduating from Collegiate High also are accepted by better universities than Exover graduates are: 40% of last year's Collegiate senior class went on to Ivy League colleges, compared to only 15% of Exover's senior class. Thus, if you want your children to get individual attention from their high school teachers and would like them to get in to good colleges, you should send them to Collegiate Prep.

In critiquing this argument, you can follow the letter writer's example and refer to Exover Academy and Collegiate Preparatory High School simply as Exover and Collegiate. You can also refer to Collegiate by its initials. Be sure, however, to identify the institution fully when you first mention it, inserting its initials in parentheses: Collegiate Preparatory High School (CPHS). Then your readers will know what you mean by future references to CPHS. Similarly, instead of typing out "for example," you can substitute the abbreviation "e.g."

TACTIC 3 Acquaint Yourself with the Actual Essay Topics You Will Face

The GRE has posted its entire selection of potential essay topics on its website. The pool of issue topics can be found at *www.gre.org/issuetop.html*. The pool of argument topics can be found at *www.gre.org/argutop.html*. There is no point in trying to memorize these topics or in trying to write an essay for each one. There are well over 200 items in the pool of issue topics alone. There is, however, a real point to exploring these potential topics and to noting their common themes.

We suggest that you print out both topic pools so that you can go through their contents at leisure. When you do so, you will see that the issue topics fall naturally into groups with common themes. Some of these themes involve contrasts:

- Tradition versus innovation and modernization.
- Competition versus cooperation.
- Present social needs versus future social needs.
- Conformity versus individualism.
- Imagination versus knowledge.
- Pragmatism versus idealism.

Many of the issue topics pose a simple question:

- What makes an effective leader?
- What are education's proper goals?
- How does technology affect our society?
- Why should we study history (or art, literature)?
- What is government's proper role (in education, art, wilderness preservation, and so on)?
- How do we define progress?

Others ask you to question conventional wisdom:

- Is loyalty *always* a virtue?
- Is "moderation in all things" *truly* good advice?
- Does conformity *always* have a negative impact?

Go over these recurrent questions and themes. They relate to all the areas of the college curriculum: political science, sociology, anthropology, economics, history, law, philosophy, psychology, the physical sciences, the fine arts, literature, even media studies. Whether or not you have any special knowledge of a suggested topic's subject area, you most likely have opinions about it. You probably have class notes on it as well.

If you have old notebooks from your general education courses, skim through them to refresh your memory of classroom discussions of typical GRE issues. In the course of flipping through these old notes, you're very likely to come across examples that you might want to note for possible use in writing the issue essay.

WRITING THE ISSUE ESSAY

TACTIC 4 Break Down the Topic Statement into Separate Areas to Consider

Here is an example of an issue topic, modeled on actual topics found in the GRE pool.

> "The end does justify the means,
> if the end is truly meritorious."

Break down the statements into its component elements. Look for key words and phrases. First, consider ***ends*** or goals. These can be divided into personal goals—taking a trip to a foreign country, for example, or providing for one's family—and societal goals—preserving endangered species, for example, or protecting the health of the elderly.

Next, consider what ***means*** you might use to reach these goals. If you have to spend your savings and take a leave of absence from college to travel abroad, thereby postponing or potentially jeopardizing your eventual graduation, then perhaps your goal is insufficiently meritorious to justify the means. If, however, your goal is not simply to take a pleasure trip but to use the time abroad working in a refugee camp, the worthiness of the cause you are serving might well outweigh the expense and the risk of your not graduating. Similarly, while most people would agree that preserving an endangered species is a worthwhile societal goal, the cost to society of doing so can occasionally outweigh the benefits: think about the societal cost in ruined crops and lost income to Klamath Basin farmers when the government cut off water to their farms in an effort to preserve endangered coho salmon and sucker fish, an action later criticized as unnecessary by the National Academy of Sciences.

Finally, consider the phrase ***truly meritorious***. The author is begging the question, qualifying his assertion to make it appear incontrovertible. But what makes an action meritorious? Even more, what makes an action *truly* meritorious? How do you measure merit? Whose standards do you use?

Breaking down the topic statement into its components helps start you thinking analytically about the subject. It's a good way to begin composing your issue essay.

TACTIC 5 Adopt a Balanced Approach

Consider your readers. Who are they? Academics, junior members of college faculties. What are they looking for? They are looking for articulate and persuasive arguments expressed in scholarly, well-reasoned prose. In other words, they are looking for the sort of essay they might write themselves.

How do you go about writing for an academic audience? First, avoid extremes. You want to come across as a mature, evenhanded writer, someone who can take a strong stand on an issue, but who can see others' positions as well. Restrain yourself: don't get so carried away by the "rightness" of your argument that you wind up sounding fanatical or shrill. Second, be sure to acknowledge that other viewpoints exist. Cite them; you'll win points for scholarly objectivity.

Draw examples to support your position from "the great world" and from the academic realm. In writing about teaching methods, for example, you'll win more points citing current newspaper articles about magnet schools or relevant passages from John Dewey and Maria Montessori than telling anecdotes about your favorite gym teacher in junior high school. While it is certainly acceptable for you to offer an occasional example from personal experience, for the most part your object is to show the readers the *breadth* of your knowledge (without showing off by quoting the most obscure sources you can find!).

One additional point: Do not try to second-guess your readers. Yes, they want you to come up with a scholarly, convincing essay. But there is no "one true answer" that they are looking for. You can argue for the position. You can argue against the position. You can strike a middle ground, arguing both for and against the position, hedging your bet. The readers don't care what position you adopt. Don't waste your time trying to psych them out.

TACTIC 6

Make Use of Transitions or Signal Words to Point the Way

TIP

Use Signal Words *Subtly*! Be Precise When You Point the Way.

Assume that typical GRE readers must read hundreds of issue essays in a day. You want to make the readers' job as easy as possible, so that when they come to your essay they breathe a sigh of relief, saying, "Ah! Someone who knows how to write!"

One way to make the readers' job easy is to lead them by the hand from one idea to the next, using signal words to point the way. The GRE readers like it when test-takers use signal words (transitions); in their analyses of sample essays scoring a 5 or 6, they particularly mention the writers' use of transitions as a good thing.

Here are a few helpful transitions. Practice using them precisely: you earn no points for sticking them in at random!

Support Signal Words

Use the following words or abbreviations to signal the reader that you are going to support your claim with an illustration or example:

e.g., (short for Latin *exempli gratia,* for the sake of an example)
for example
for instance
let me illustrate
such as

Use these words to signal the reader that you are about to add an additional reason or example to support your claim:

additionally	*furthermore*	*likewise*
also	*in addition*	*moreover*

Contrast Signal Words

Use the following words to signal a switch of direction in your argument.

although	*in contrast*	*on the other hand*
but	*in spite of*	*rather than*
despite	*instead of*	*still*
even though	*nevertheless*	*unlike*
except	*not*	*yet*
however	*on the contrary*	

Cause and Effect Signal Words

Use the following words to signal the next step in your line of reasoning or the conclusion of your argument.

accordingly	*in conclusion*	*therefore*
consequently	*in short*	*thus*
for this reason	*in summary*	*when . . . then*
hence	*so . . . that*	

See Tactic 10 for a discussion of how signal words can be helpful to you in the second of your two writing tasks, the argument critique.

WRITING THE ARGUMENT CRITIQUE

TACTIC 7

Learn to Spot Common Logical Fallacies

You may remember studying a list of logical fallacies during your undergraduate education. It probably included Latin terms such as "post hoc ergo propter hoc" and "argumentum ad hominem." Fortunately, you do not need to memorize these terms to perform well on the GRE argument essay. The GRE's essay readers are not concerned with whether you know the name of a given logical fallacy; they are more concerned with whether you can recognize and explain fallacies as they occur in simulated real-world situations. Labeling a claim a "post hoc" fallacy will not win you a 6 (the top score) unless you can *explain* the flaw in the argument. And a straightforward logical explanation of the argument's flaw can get you a 6, whether or not you use the fancy Latin terminology.

This does not mean, however, that brushing up on the common logical fallacies is a waste of your time. A decent understanding of the ways in which arguments can be wrong will help you write a better essay by enabling you to identify more flaws

in the assigned argument (GRE argument statements generally include more than one logical error), and by giving you a clearer understanding of the nature of those flaws. Our advice is, therefore, to review the common logical fallacies without spending too much time trying to memorize their names.

Here are two examples of arguments, or prompts, similar to those in the GRE pool. Read them. The discussion following will point out what common logical fallacies they embody.

Discuss how effective you find the reasoning in this argument.

ARGUMENT 1

The school board of the Shadow Valley Unified School District included these remarks in a letter sent to the families of all students attending school in the district.

> Over the past few years, an increase in disciplinary problems and a high drop out rate have plagued District schools. The Ash Lake School District to our north adopted a mandatory uniform policy three years ago. Since that time, suspensions and expulsions in Ash Lake have fallen by 40 percent, while the mean grade point average of Ash Lake students has risen from 2.3 (C+) to 2.7 (B−). In order to improve the discipline and academic performance of Shadow Valley students, we have adopted a mandatory uniform policy effective on the first day of the new school year.

Discuss how effective you find the reasoning in this argument.

ARGUMENT 2

The following is excerpted from a letter to the editor in the Chillington Gazette.

> The recent residential property tax increase to improve park maintenance in Chillington is a waste of money. There is no need to improve Chillington's parks because the people of Chillington do not enjoy outdoor recreation. I live across the street from Green Park in South Chillington, and I've noticed that there is never anyone in the park. Park use did not increase in Warm Springs last year when they implemented a similar tax. There is no reason to improve parks that will not be used.

COMMON LOGICAL FALLACIES

CAUSAL FALLACIES

The classic fallacy of causation is often known by a Latin phrase, "post hoc ergo propter hoc," or its nickname, "the post hoc fallacy." The Latin phrase translates to, "after this, therefore because of this." The post hoc fallacy confuses correlation with causation, assuming that when one event follows another, the second event must have been caused by the first. It is as if you were to say that because your birthday precedes your husband's by one month, your birth must have caused him to be born. The Shadow Valley School District argument presents an excellent example of a post hoc fallacy. The author of this argument assumes that because "suspensions and expulsions in Ash Lake have fallen by 40 percent, while the mean grade point average of Ash Lake students has risen from 2.3 (C+) to 2.7 (B−)" since Ash Lake's adoption of a mandatory uniform policy, the uniform policy has caused the improved student performance. Despite this correlation, it is possible that other factors are responsible for Ash Lake's progress. Perhaps the school uniform policy coincided with a significant decrease in average class size, or the arrival of a new superintendent of schools. Or perhaps the recent improvements were brought about by an increase in federal aid for at-risk students. School uniforms may have been a partial cause of Ash Lake's improvements, or they may have played no role at all. Without further information, no reliable conclusion can be reached.

INDUCTIVE FALLACIES

Fallacies of induction involve the drawing of general rules from specific examples. They are among the most common fallacies found in the GRE argument essay topics. To induce a general rule correctly from specific examples, it is crucial that the specific examples be representative of the larger group. All too often, this is not the case.

The **hasty generalization** (too small sample) is the most common inductive fallacy. A hasty generalization is a general conclusion that is based on too small a sample set. If, for example, you wanted to learn the most popular flavor of ice cream in Italy, you would need to interview a substantial number of Italians. Drawing a conclusion based on the taste of the three Italian tourists you met last week would not be justified. The *Chillington Gazette* argument provides another good example of the hasty generalization. The author of this argument concludes that "the people of Chillington do not enjoy outdoor recreation," but he draws this general conclusion from the lack of visitors to the park across the street from his home. Readers are never told just how many parks there are in Chillington. There could be dozens of parks, all possibly overflowing with happy visitors, despite the unpopularity of the one park viewed by the author.

Small sample size is a problem because it increases the risk of drawing a general conclusion from an **unrepresentative sample**. If, for example, you wanted to learn who was most likely to be elected president of the United States, you could not draw a reliable conclusion based on the preferences of the citizens of a single city, or even a single state. The views of the citizens of Salt Lake City are not necessarily the views of the citizens of the nation as a whole, nor are the views of Californians representative of those of the entire nation. This is why pollsters go to such great lengths to ensure that they interview a representative sample of the entire population.

Unrepresentative samples do not, however, always result from too small a sample. The *Chillington Gazette* argument concludes that the citizens of Chillington will not use improved parks because "Park use did not increase in Warm Springs last year when they implemented a similar tax." The author gives no reason to believe, however, that the two towns' situations are similar. Perhaps park use did not increase in Warm Springs because its parks were already extremely popular, unlike those of

Chillington. Or perhaps Warm Springs is an industrial city with little housing, while Chillington is a bedroom community with a large number of school-aged children. Should we conclude that the experiences of one city will be mirrored by the other?

(To learn more about common logical fallacies, consult standard works on rhetoric and critical reasoning. Two currently popular texts are James Herrick's *Argumentation* and T. Edward Damer's *Attacking Faulty Reasoning*.)

TACTIC 8 Remember That Your Purpose Is to Analyze, *Not* to Persuade

You are not asked to agree or disagree with the argument in the prompt. Do not be distracted by your feelings on the subject of the prompt, and do not give in to the temptation to write your own argument. Be especially vigilant against this temptation if the topic is on a subject that you know very well. If, for example, the prompt argues that class size reduction is a poor idea because it did not improve test scores in one city, do not answer this argument with data you happen to know about another city in which test scores improved after class sizes were reduced. Instead, point out that one city is not a large enough sample on which to base a general conclusion. Go on to identify other factors that could have caused test scores to remain the same, despite lower class size. (Perhaps test scores in the sample city were already nearly as high as they could go, or the student population in that city was changing at the time class sizes were reduced.) Remember, the readers are not interested in how much you *know* about the subject of the prompt; they want to know how well you *think*.

TACTIC 9 Examine the Argument for Unstated Assumptions and Missing Information

An argument is based upon certain assumptions made by its author. If an argument's basic premises are sound, the argument is strengthened. If the argument's basic premises are flawed, the argument is weakened.

Pinpoint what the argument assumes but never states. Then consider the validity of these unstated assumptions. For example, the Shadow Valley argument assumes that the populations of Shadow Valley and Ash Lake are analogous. Is this unstated assumption warranted? Not necessarily. The two towns might well have distinctly dissimilar populations—one might be a working-class suburb with high unemployment, while the other might be a suburb populated by wealthy professionals. If that were so, there would be no reason to believe that the same factors would cause poor student performance in both towns.

Ask yourself what additional evidence would strengthen or weaken the claim. Generally, GRE argument prompts are flawed but could be true under some circumstances. Only rarely will you find an argument that is absolutely untrue. Instead, you will find plausible arguments for which support (grounds and warrant) is lacking. Put yourself in the place of the argument's author. If you were trying to prove this argument, what evidence would you need? What missing data should you assemble to support your claim? Use your concluding paragraph to list this evidence and explain how its presence would solve the shortcomings that you identified earlier in your essay.

TACTIC 10 Pay Particular Attention to Signal Words in the Argument

In analyzing arguments, be on the lookout for transitions or signal words that can clarify the structure of the argument. These words are like road signs, pointing out the direction the author wants you to take, showing you the connection between one logical step and the next. When you spot such a word linking elements in the author's argument, ask yourself whether this connection is logically watertight. Does A unquestionably lead to B? These signal words can indicate vulnerable areas in the argument, points you can attack.

In particular, be alert for:

Cause and Effect Signal Words

The following words often signal the conclusion of an argument.

accordingly	*in conclusion*	*therefore*
consequently	*in short*	*thus*
for this reason	*in summary*	
hence	*so*	

Contrast Signal Words

The following words often signal a reversal of thought within an argument.

although	*in contrast*	*rather than*
but	*instead*	*still*
despite	*nevertheless*	*unlike*
even though	*not*	*yet*
except	*on the contrary*	
however	*on the other hand*	

Notice that in the following argument several of these words are present: *despite*, *not*, and *consequently*. Each of these words plays an important role in the argument.

Discuss how effective you find the reasoning in this argument.

ARGUMENT 3

The following is from a letter to the state Department of Education.

> Despite the fact that the River City School District increased the average size by more than 15% in all grades two years ago, this year's average SAT scores for the junior class were the highest ever. This shows that class size is not a good determinant of student performance. Consequently, other school districts should follow River City's lead and save money by increasing the size of their classes.

Think about each link in the chain of reasoning signaled by the three transition words. These words should act like a red flag, alerting you that danger (flawed logic) may lie ahead. Did the average SAT score for the junior class increase *despite* the increase in class size? Maybe. Then again, maybe not; the average score for that year's junior class may have increased because that year's juniors were unusually bright. Do this year's extra-high SAT scores show that class size is *not* a good determinant of student performance? Not necessarily. Many factors could have contributed to the junior class's high scores. Finally, consider the implications of *consequently.* Even if class size were not a good determinant of student performance, does it necessarily follow *as a consequence* that school districts should increase the size of their classes? In the words of the old song, "It ain't necessarily so."

Practice Exercises

Practice for the Issue Task

1. Brainstorm for 5 minutes, jotting down any words and phrases that are triggered by one of the following questions:

   - What should the goals of higher education be?
   - Why should we study history?
   - How does technology affect our society?
   - What is the proper role of art?
   - Which poses the greater threat to society, individualism or conformity?
   - Which is more socially valuable, preserving tradition or promoting innovation?
   - Is it better to be a specialist or a generalist?
   - Can a politician be both honest and effective?

2. In a brief paragraph, define one of the following words:

   - Freedom
   - Originality
   - Honesty
   - Progress

3. To improve your ear for language, read aloud short selections of good prose: editorials from *The New York Times* or *The Christian Science Monitor*, as well as columns or brief essays by prose stylists like Annie Dillard, M. F. K. Fisher, or E. B. White. Listen for the ways in which these authors vary their sentence structure. Note the precision with which they choose their words. The more good prose you hear, the better able you'll be to improve your writing style.

4. Selecting three or four issue topics from the GRE's online pool of topics for the revised test (currently at *http://www.ets.org/gre/revised_general/prepare/analytical_writing/issue/pool)*, break down the topic statements in terms of Toulmin's three elements: claim, grounds, and warrant. Ask yourself the following questions. What claims are made in each topic statement? What grounds or data are given to support each of these claims? Is the claim warranted or unwarranted? Why? In what way do the grounds logically justify the claim?

5. Choosing another issue topic from the GRE's published pool of topics, write an essay giving your viewpoint concerning the particular issue raised. Set no time limit; take as long as you want to complete this task, then choose a second issue topic from the pool. *In only 30 minutes*, write an essay presenting your perspective on this second issue.

   Compare your two essays. Ask yourself how working under time pressure affected your second essay. Did its major problems stem from a lack of fluency? A lack of organization? A lack of familiarity with the subject matter under discussion? A lack of knowledge of the mechanics of formal written English? Depending on what problems you spot, review the appropriate sections of this chapter, as well as any style manuals or other texts we suggest.

Practice for the Argument Task

1. Choosing a sample of argument topics from GRE's online pool of topics for the revised test (currently at *http://www.ets.org/gre/revised_general/prepare/analytical_writing/argument/pool*), practice applying the list of logical fallacies to the published prompts. See how many fallacies you can find for each argument. If you have time, write practice essays for some of these arguments. If you are short of time, or would simply like to move more quickly, get together with a friend and explain the fallacies you have found in the argument essay prompts. This will be especially rewarding if you can work with a friend who is also preparing to take the GRE.

2. Write an "original" argument topic, modeling it on one of the argument prompts in the GRE's published pool. Your job is to change the details of the situation (names, figures, and so on) without changing the types of logical fallacies involved. By doing this, you will learn to spot the same old fallacies whenever they crop up in a new guise.

3. Choosing an argument prompt from GRE's online pool of topics for the revised test (currently at *http://www.ets.org/gre/revised_general/prepare/analytical_writing/argument/pool*), write an essay critiquing the particular argument expressed. Set no time limit; take as long as you want to complete this task, then choose a second argument prompt from the pool. *In only 30 minutes*, write an essay critiquing this second argument.

   Compare your two critiques. Ask yourself how working under time pressure affected your second critique. Would more familiarity with the common logical fallacies have helped you? Depending on what problems you spot, review the appropriate sections of this chapter, as well as any other materials we suggest.

PART 4

QUANTITATIVE ABILITY: TACTICS, STRATEGIES, PRACTICE, AND REVIEW

Introduction to Part 4

Part 4 consists of five chapters. Chapter 7 presents several important strategies that can be used on any mathematics questions that appear on the GRE. In Chapters 8, 9, and 10 you will find tactics that are specific to one of the three different types of questions: discrete quantitative questions, quantitative comparison questions, and data interpretation questions, respectively. Chapter 11 contains a complete review of all the mathematics you need to know in order to do well on the GRE, as well as hundreds of sample problems patterned on actual test questions.

FIVE TYPES OF TACTICS

Five different types of tactics are discussed in this book.

1. In Chapters 1 and 2, you learned many basic tactics used by all good test-takers, such as read each question carefully, pace yourself, don't get bogged down on any one question, and never waste time reading the directions. You also learned the specific tactics required to excel on a computerized test. These tactics apply to both the verbal and quantitative sections of the GRE.
2. In Chapters 4 and 5 you learned the important tactics needed for handling each type of verbal question.
3. In Chapter 6 you learned the strategies for planning and writing the two essays that constitute the analytical writing section of the GRE.

4. In Chapters 7–10 you will find all of the tactics that apply to the quantitative sections of the GRE. Chapter 7 contains those techniques that can be applied to all types of mathematics questions; Chapters 8, 9, and 10 present specific strategies to deal with each of the three kinds of quantitative questions found on the GRE: discrete quantitative questions, quantitative comparison questions, and data interpretation questions.
5. In Chapter 11 you will learn or review all of the mathematics that is needed for the GRE, and you will master the specific tactics and key facts that apply to each of the different mathematical topics.

Using these tactics will enable you to answer more quickly many questions that you already know how to do. But the greatest value of these tactics is that they will allow you to correctly answer or make educated guesses on problems that *you do not know how to do.*

WHEN TO STUDY CHAPTER 11

How much time you initially devote to Chapter 11 should depend on how good your math reasoning skills are, how long it has been since you studied math, and how much of the math you learned in middle school and the first two years of high school you remember. If you think that your math skills are quite good, you can initially skip the instructional parts of Chapter 11. If, however, after doing the Model Tests in Part 5 of this book, you find that you made more than one or two mistakes on questions involving the same topic (averages, percents, geometry, etc.) or you spent too much time on them, you should then study the appropriate sections of Chapter 11. Even if your math skills are excellent, you should do the exercises in Chapter 11; they are a good source of additional GRE questions. If your math skills were never very good or if you feel they are rusty, it is advisable to review the material in Chapter 11, including working out the problems, *before* tackling the Model Tests.

AN IMPORTANT SYMBOL

Throughout the rest of this book, the symbol "⇒" is used to indicate that one step in the solution of a problem follows *immediately* from the preceding one, and no explanation is necessary. You should read

$3x = 12 \Rightarrow x = 4$

as $3x = 12$ *implies that* $x = 4$

or $3x = 12$, *which implies that* $x = 4$

or *since* $3x = 12$, *then* $x = 4$.

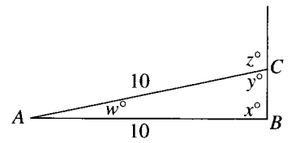
Here is a sample solution to the following problem using ⇒:

What is the value of $2x^2 - 5$ when $x = -4$?

$x = -4 \Rightarrow x^2 = (-4)^2 = 16 \Rightarrow 2x^2 = 2(16) = 32 \Rightarrow 2x^2 - 5 = 32 - 5 = \mathbf{27}$

When the reason for a step is not obvious, ⇒ is not used: rather, an explanation is given, often including a reference to a KEY FACT from Chapter 11. In many solutions, some steps are explained, while others are linked by the ⇒ symbol, as in the following example.

In the diagram below, if $w = 10$, what is the value of z?

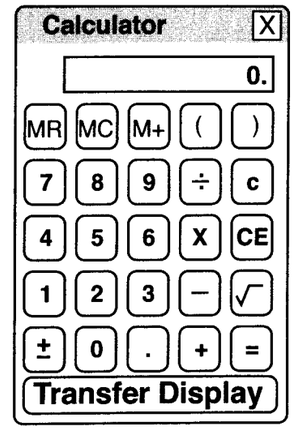


- By KEY FACT J1, $w + x + y = 180$.
- Since $\triangle ABC$ is isosceles, $x = y$ (KEY FACT J5).
- Therefore, $w + 2y = 180 \Rightarrow 10 + 2y = 180 \Rightarrow 2y = 170 \Rightarrow y = 85$.
- Finally, since $y + z = 180$ (KEY FACT I3), $85 + z = 180 \Rightarrow z = \mathbf{95}$.

CALCULATORS ON THE GRE

You may *not* bring your own calculator to use when you take the GRE. However, starting in 2011, for the first time ever, you will have access to an onscreen calculator. While you are working on the math sections, one of the icons at the top of the screen will be a calculator icon. During the verbal and writing sections of the test, either that icon will be greyed out (meaning that you can't click on it) or it will simply not be there at all. During the math sections, however, you will be able to click on that icon at anytime; when you do, a calculator will instantly appear on the screen. Clicking the X in the upper-right-hand corner of the calculator will hide it.

Note that when the calculator appears on the screen, it may cover part of the question or the answer choices. If this occurs, just click on the top of the calculator and drag it to a convenient location. If you use the calculator to answer a question and then click NEXT to go to the next question, the calculator remains on the screen, exactly where it was, with the same numerical readout. This is actually a distraction. So, if you do use the calculator to answer a question, as soon as you have answered that question, click on the X to remove the calculator from the screen. Later, it takes only one click to get it back.



The onscreen calculator is a simple four-function calculator, with a square root key. It is not a graphing calculator; it is not a scientific calculator. The only operations you can perform with the onscreen calculator are adding, subtracting, multiplying, dividing, and taking square roots. Fortunately, these are the only operations you will ever need to answer any GRE question.

At the bottom of the onscreen calculator is a bar labeled TRANSFER DISPLAY. If you are using the calculator on a numeric entry question, and the result of your final calculation is the answer that you want to enter in the box, click on TRANSFER DISPLAY —the number currently displayed in the calculator's readout will instantly appear in the box under the question. This saves the few seconds that it would otherwise take to enter your answer; more important, it guarantees that you won't make an error typing in your answer.

Just because you have a calculator at your disposal does not mean that you should use it very much. In fact, you shouldn't. The vast majority of questions that appear on the GRE do not require any calculations.

Remember

Use your calculator only when you need to.

CHAPTER 7

General Math Strategies

In Chapters 8 and 9, you will learn tactics that are specifically applicable to discrete quantitative questions and quantitative comparison questions, respectively. In this chapter you will learn several important general math strategies that can be used on both of these types of questions.

The directions that appear on the screen at the beginning of the quantitative sections include the following cautionary information:

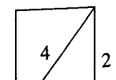
> Figures that accompany questions are intended to provide information useful in answering the questions.
>
> However, unless a note states that a figure is drawn to scale, you should solve these problems NOT by estimating sizes by sight or measurement, but by using your knowledge of mathematics.

Despite the fact that they are telling you that you cannot totally rely on *their* diagrams, if you learn how to draw diagrams accurately, *you can trust the ones you draw.* Knowing the best ways of handling diagrams on the GRE is critically important. Consequently, the first five tactics all deal with diagrams.

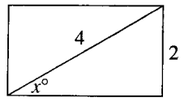
TACTIC 1.	**Draw a diagram.**
TACTIC 2.	**Trust a diagram that has been drawn to scale.**
TACTIC 3.	**Exaggerate or change a diagram.**
TACTIC 4.	**Add a line to a diagram.**
TACTIC 5.	**Subtract to find shaded regions.**

To implement these tactics, you need to be able to draw line segments and angles accurately, and you need to be able to look at segments and angles and accurately estimate their measures. Let's look at three variations of the same problem.

1. If the diagonal of a rectangle is twice as long as the shorter side, what is the degree measure of the angle it makes with the longer side?
2. In the rectangle below, what is the value of x?



3. In the rectangle below, what is the value of x?



For the moment, let's ignore the correct mathematical way of solving this problem. In the diagram in (3), the side labeled 2 appears to be half as long as the diagonal, which is labeled 4; consequently, you should assume that the diagram has been drawn to scale, and you should see that x is about 30, *certainly* between 25 and 35. In (1) you aren't given a diagram, and in (2) the diagram is useless because you can see that it has not been drawn to scale (the side labeled 2 is nearly as long as the diagonal, which is labeled 4). However, if while taking the GRE, you see a question such as (1) or (2), you should be able to quickly draw on your scrap paper a diagram that looks just like the one in (3), and then look at *your* diagram and see that the measure of x is just about 30. If the answer choices for these questions were

Ⓐ 15 Ⓑ 30 Ⓒ 45 Ⓓ 60 Ⓔ 75

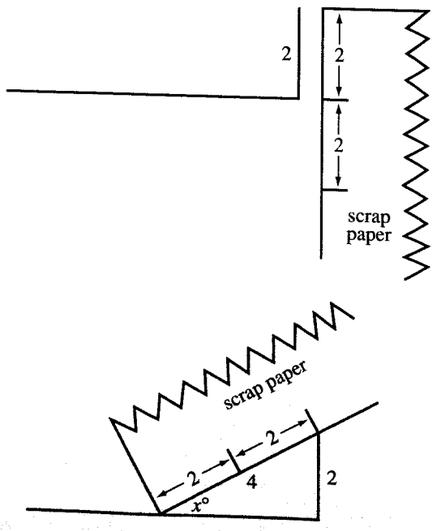
you would, of course, choose **30**, **B**. If the choices were

Ⓐ 20 Ⓑ 25 Ⓒ 30 Ⓓ 35 Ⓔ 40

you might not be quite as confident, but you should still choose **30**, here **C**.

When you take the GRE, even though you are not allowed to have rulers or protractors, you should be able to draw your diagrams very accurately. For example, in (1) above, you should draw a horizontal line, and then, either freehand or by tracing the corner of a piece of scrap paper, draw a right angle on the line. The vertical line segment will be the width of the rectangle; label it 2.

Mark off that distance twice on a piece of scrap paper and use that to draw the diagonal.

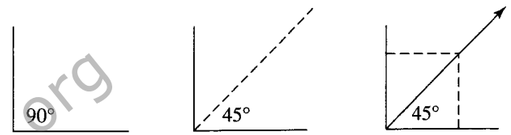


You should now have a diagram that is similar to that in (3), and you should be able to see that x is about 30.

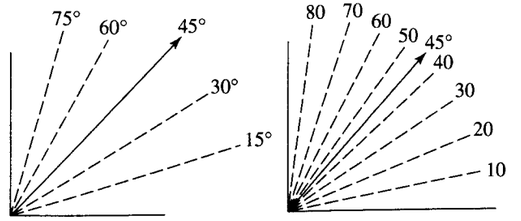
By the way, x is *exactly* 30. A right triangle in which one leg is half the hypotenuse must be a 30-60-90 triangle, and that leg is opposite the 30° angle [see KEY FACT J11].

Having drawn an accurate diagram, are you still unsure as to how you should know that the value of x is 30 just by looking at the diagram? You will now learn not only how to look at *any* angle and know its measure within 5 or 10 degrees, but how to draw any angle that accurately.

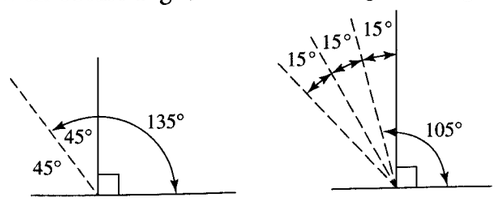
You should easily recognize a 90° angle and can probably draw one freehand; but you can always just trace the corner of a piece of scrap paper. To draw a 45° angle, just bisect a 90° angle. Again, you can probably do this freehand. If not, or to be even more accurate, draw a right angle, mark off the same distance on each side, draw a square, and then draw in the diagonal.



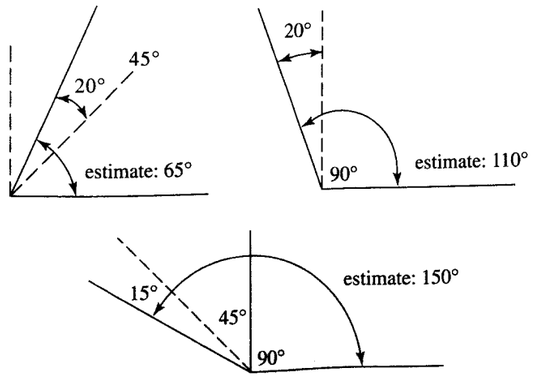
To draw other acute angles, just divide the two 45° angles in the above diagram with as many lines as necessary.



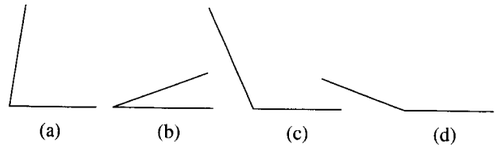
Finally, to draw an obtuse angle, add an acute angle to a right angle.



Now, to estimate the measure of a given angle, just draw in some lines.



To test yourself, find the measure of each angle shown. The answers are found below.



Answers (a) 80° (b) 20° (c) 115° (d) 160°. Did you come within 10° on each one?

Testing Tactics

TACTIC 1 Draw a Diagram

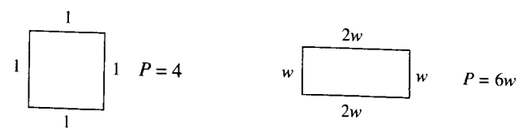
On *any* geometry question for which a figure is not provided, draw one (as accurately as possible) on your scrap paper — *never attempt a geometry problem without first drawing a diagram.*

EXAMPLE 1

What is the area of a rectangle whose length is twice its width and whose perimeter is equal to that of a square whose area is 1?

Ⓐ 1 Ⓑ 6 Ⓒ $\frac{2}{3}$ Ⓓ $\frac{4}{3}$ Ⓔ $\frac{8}{9}$

SOLUTION. Don't even think of answering this question until you have drawn a square and a rectangle and labeled each of them: each side of the square is 1, and if the width of the rectangle is w, its length (ℓ) is $2w$.



Now, write the required equation and solve it:

$$6w = 4 \Rightarrow w = \frac{4}{6} = \frac{2}{3} \Rightarrow 2w = \frac{4}{3}$$

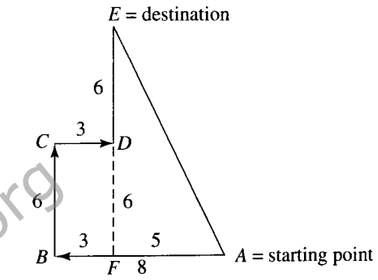
The area of the rectangle $= \ell w = \left(\frac{4}{3}\right)\left(\frac{2}{3}\right) = \frac{8}{9}$, **E.**

EXAMPLE 2

Betty drove 8 miles west, 6 miles north, 3 miles east, and 6 more miles north. How many miles was Betty from her starting place?

☐ miles

SOLUTION. Draw a diagram showing Betty's route from A to B to C to D to E.



Now, extend line segment ED until it intersects AB at F. Then, AFE is a right triangle, whose legs are 5 and 12. The length of hypotenuse AE represents the distance from her starting point to her destination. Either recognize that $\triangle AFE$ is a 5-12-13 right triangle or use the Pythagorean theorem:

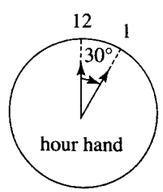
$$5^2 + 12^2 = (AE)^2 \Rightarrow (AE)^2 = 25 + 144 = 169 \Rightarrow AE = \mathbf{13}.$$

EXAMPLE 3

What is the difference in the degree measures of the angles formed by the hour hand and the minute hand of a clock at 12:35 and 12:36?

Ⓐ 1° Ⓑ 5° Ⓒ 5.5° Ⓓ 6° Ⓔ 30°

SOLUTION. Draw a simple picture of a clock. The hour hand makes a complete revolution, 360°, once every 12 hours. So, in 1 hour it goes through 360° ÷ 12 = 30°, and in one minute it advances through 30° ÷ 60 = 0.5°. The minute hand moves through 30° every 5 minutes or 6° per minute. So, in the minute from 12:35 to 12:36 (or any other minute), the *difference* between the hands increased by 6° – 0.5° = **5.5°**, **C.**



30° in 1 hr.
0.5° in 1 min.

30° in 5 min.
6° in 1 min.

NOTE: It was not necessary, and would have been more time-consuming, to determine the angle between the hands at either 12:35 or 12:36. (See TACTIC 6: Don't do more than you have to.)

Drawings should not be limited to geometry questions; there are many other questions on which drawings will help.

EXAMPLE 4

A jar contains 10 red marbles and 30 green ones. How many red marbles must be added to the jar so that 60% of the marbles will be red?

SOLUTION. Let x represent the number of red marbles to be added, and draw a diagram and label it.

x	Red
30	Green
10	Red

From the diagram it is clear that there are now $40 + x$ marbles in the jar, of which $10 + x$ are red. Since we want the fraction of red marbles to be 60%, we have $\frac{10+x}{40+x} = 60\% = \frac{60}{100} = \frac{3}{5}$. Cross-multiplying, we get:

$$5(10 + x) = 3(40 + x) \Rightarrow 50 + 5x = 120 + 3x \Rightarrow 2x = 70 \Rightarrow x = \mathbf{35}.$$

Of course, you could have set up the equation and solved it without the diagram, but the diagram makes the solution easier and you are less likely to make a careless mistake.

TACTIC 2 Trust a Diagram That Has Been Drawn to Scale

Whenever diagrams have been drawn to scale, they can be trusted. This means that you can look at the diagram and use your eyes to accurately estimate the sizes of angles and line segments. For example, in the first problem discussed at the beginning of this chapter, you could "see" that the measure of the angle was about 30°.

To take advantage of this situation:

- If a diagram is given that appears to be drawn to scale, trust it.
- If a diagram is given that has not been drawn to scale, try to draw it to scale on your scrap paper, and then trust it.
- When no diagram is provided, and you draw one on your scrap paper, try to draw it to scale.

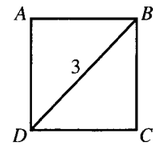
In Example 5 below, we are told that $ABCD$ is a square and that diagonal BD is 3. In the diagram provided, quadrilateral $ABCD$ does indeed look like a square, and

$BD = 3$ does not contradict any other information. We can, therefore, assume that the diagram has been drawn to scale.

EXAMPLE 5

In the figure at the right, diagonal BD of square $ABCD$ is 3. What is the perimeter of the square?

Ⓐ 4.5 Ⓑ 12 Ⓒ $3\sqrt{2}$ Ⓓ $6\sqrt{2}$ Ⓔ $12\sqrt{2}$



SOLUTION. Since this diagram has been drawn to scale, you can trust it. The sides of the square appear to be about two thirds as long as the diagonal, so assume that each side is about 2. Then the perimeter is about 8. Which of the choices is approximately 8? Certainly not A or B. Since $\sqrt{2} \approx 1.4$, Choices C, D, and E are approximately 4.2, 8.4, and 12.6, respectively. Clearly, the answer must be **D**.

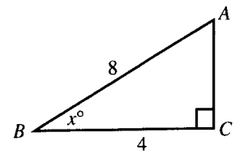
Direct mathematical solution. Let s be a side of the square. Then since $\triangle BCD$ is a 45-45-90 right triangle, $s = \frac{3}{\sqrt{2}} = \frac{3\sqrt{2}}{2}$, and the perimeter of the square is $4s = 4\left(\frac{3\sqrt{2}}{2}\right) = 6\sqrt{2}$.

Remember the goal of this book is to help you get credit for *all* the problems you know how to do, and, by using the TACTICS, to get credit for *many* that you don't know how to do. Example 5 is typical. Many students would miss this question. *You*, however, can now answer it correctly, even though you may not remember how to solve it directly.

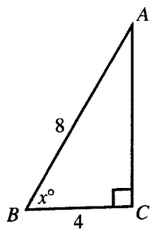
EXAMPLE 6

In $\triangle ABC$, what is the value of x?

Ⓐ 75 Ⓑ 60 Ⓒ 45 Ⓓ 30 Ⓔ 15



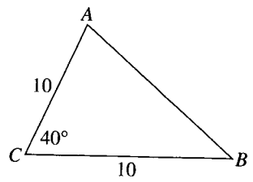
SOLUTION. If you don't see the correct mathematical solution, you should use TACTIC 2 and trust the diagram; but to do that you must be careful that when you copy it onto your scrap paper you *fix it*. What's wrong with the way it is drawn now? $AB = 8$ and $BC = 4$, but in the figure, AB and BC are almost the same length. Redraw it so that AB is *twice* as long as BC. Now, just look: x is about **60**, **B**.



In fact, x is exactly 60. If the hypotenuse of a right triangle is twice the length of one of the legs, then it's a 30-60-90 triangle, and the angle formed by the hypotenuse and that leg is 60° (see Section 11-J).

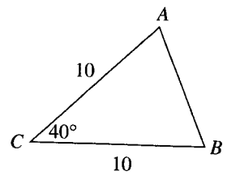
TACTIC 2 is equally effective on quantitative comparison questions that have diagrams. See pages 9–11 for directions on how to solve quantitative comparison questions.

EXAMPLE 7



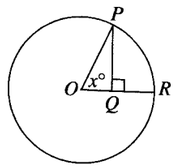
Quantity A	Quantity B
AB	10

SOLUTION. There are two things wrong with the given diagram: $\angle C$ is labeled 40°, but looks much more like 60° or 70°, and AC and BC are each labeled 10, but BC is drawn much longer. When you copy the diagram onto your scrap paper, be sure to correct these two mistakes: draw a triangle that has a 40° angle and two sides of the same length.



Now, it's clear: $AB < 10$. The answer is **B**.

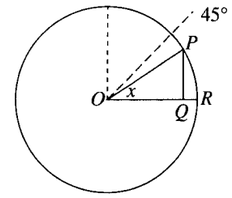
EXAMPLE 8



O is the center of the circle
$PQ = 6$
$OR = 12$

Quantity A	Quantity B
x	45

SOLUTION. In the diagram on page 154, the value of x is at least 60, so if the diagram has been drawn to scale, the answer would be **A**. If, on the other hand, the diagram has not been drawn to scale, we can't trust it. Which is it? The diagram is *not* drawn to scale — PQ is drawn almost as long as OR, even though OR is twice as long. Correct the diagram:

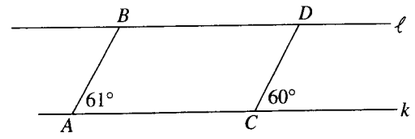


Now you can see that x is less than 45. The answer is **B**.

TACTIC 3 Exaggerate or Otherwise Change a Diagram

Sometimes it is appropriate to take a diagram that appears to be drawn to scale and intentionally exaggerate it. Why would we do this? Consider the following example.

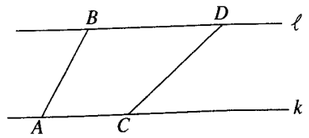
EXAMPLE 9



Line ℓ is parallel to line *k*.

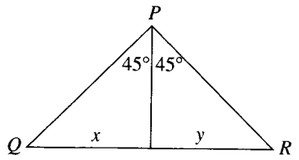
Quantity A	Quantity B
AB	*CD*

SOLUTION. In the diagram, which appears to be drawn correctly, AB and CD look as though they are the same length. However, there *might* be an imperceptible difference due to the fact that angle C is slightly smaller than angle A. So exaggerate the diagram: redraw it, making angle C *much* smaller than angle A. Now, it's clear: CD is longer. The answer is **B**.



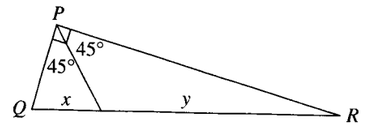
When you copy a diagram onto your scrap paper, you can change anything you like as long as your diagram is consistent with all the given data.

EXAMPLE 10



Quantity A	Quantity B
x	y

SOLUTION. You may redraw this diagram any way you like, as long as the two angles that are marked 45° remain 45°. If PQ and PR are equal, as they appear to be in the given diagram, then x would equal y. Since the given information doesn't state that $PQ = PR$, draw a diagram in which PQ and PR are clearly unequal. In the diagram below, PR is much longer than PQ, and x and y are clearly unequal. The answer is **D**.



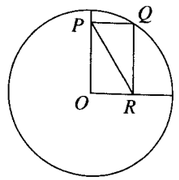
TACTIC

4 **Add a Line to a Diagram**

Occasionally, after staring at a diagram, you still have no idea how to solve the problem to which it applies. It looks as though not enough information has been given. When this happens, it often helps to draw another line in the diagram.

EXAMPLE 11

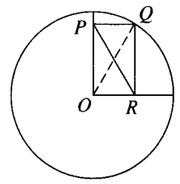
In the figure below, Q is a point on the circle whose center is O and whose radius is r, and $OPQR$ is a rectangle. What is the length of diagonal PR?



Ⓐ r Ⓑ r^2 Ⓒ $\frac{r^2}{\pi}$ Ⓓ $\frac{r\sqrt{2}}{\pi}$

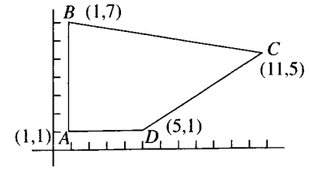
Ⓔ It cannot be determined from the information given.

SOLUTION. If after staring at the diagram and thinking about rectangles, circles, and the Pythagorean theorem, you're still lost, don't give up. Ask yourself, "Can I add another line to this diagram?" As soon as you think to draw in OQ, the other diagonal, the problem becomes easy: the two diagonals of a rectangle have the same length and, since OQ is a radius, it is equal to r, **A**.

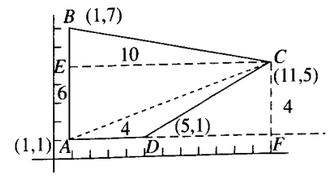


EXAMPLE 12

What is the area of quadrilateral $ABCD$?

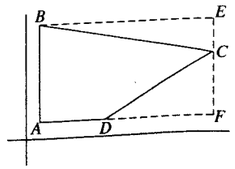


SOLUTION. Since the quadrilateral is irregular, there isn't a formula to find the area. However, if you draw in AC, you will divide $ABCD$ into two triangles, each of whose areas can be determined.



If you then draw in the height of each triangle, you see that the area of $\triangle ACD$ is $\frac{1}{2}(4)(4) = 8$, and the area of $\triangle BAC$ is $\frac{1}{2}(6)(10) = 30$, so the area of $ABCD$ is $30 + 8 =$ **38**.

Note that this problem could also have been solved by drawing in lines to create rectangle $ABEF$, and subtracting the areas of $\triangle BEC$ and $\triangle CFD$ from the area of the rectangle.

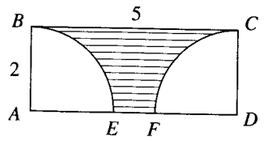


TACTIC 5 Subtract to Find Shaded Regions

Whenever part of a figure is shaded, the straightforward way to find the area of the shaded portion is to find the area of the entire figure and subtract from it the area of the unshaded region. Of course, if you are asked for the area of the unshaded region, you can, instead, subtract the shaded area from the total area. Occasionally, you may see an easy way to calculate the shaded area directly, but usually you should subtract.

EXAMPLE 13

In the figure below, *ABCD* is a rectangle, and *BE* and *CF* are arcs of circles centered at *A* and *D*. What is the area of the striped region?

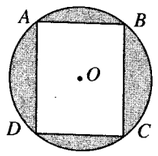


Ⓐ $10 - \pi$ Ⓑ $2(5 - \pi)$ Ⓒ $2(5 - 2\pi)$ Ⓓ $6 + 2\pi$ Ⓔ $5(2 - \pi)$

SOLUTION. The entire region is a 2×5 rectangle whose area is 10. Since the white region consists of two quarter-circles of radius 2, the total white area is that of a semicircle of radius 2: $\frac{1}{2}\pi(2)^2 = 2\pi$. Therefore, the area of the striped region is $10 - 2\pi = \mathbf{2(5 - \pi)}$, **B**.

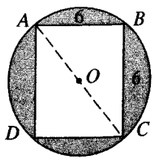
EXAMPLE 14

In the figure below, square *ABCD* is inscribed in circle *O*. If the perimeter of *ABCD* is 24, what is the area of the shaded region?



Ⓐ $18\pi - 36$ Ⓑ $18\pi - 24$ Ⓒ $12\pi - 36$ Ⓓ $9\pi - 36$ Ⓔ $9\pi - 24$

SOLUTION. Since the perimeter of square *ABCD* is 24, each of its sides is 6, and its area is $6^2 = 36$. Since diagonal *AC* is the hypotenuse of isosceles right triangle *ABC*, $AC = 6\sqrt{2}$. But *AC* is also a diameter of circle *O*, so the radius of the circle is $3\sqrt{2}$, and its area is $\pi(3\sqrt{2})^2 = 18\pi$. Finally, the area of the shaded region is $\mathbf{18\pi - 36}$, **A**.



TACTIC 6 Don't Do More Than You Have To

Very often a problem can be solved in more than one way. You should always try to do it in the easiest way possible. Consider the following examples.

EXAMPLE 15

If $5(3x - 7) = 20$, what is $3x - 8$?

Ⓐ $\frac{11}{3}$ Ⓑ 0 Ⓒ 3 Ⓓ 14 Ⓔ 19

It is not difficult to solve for x:

$$5(3x - 7) = 20 \Rightarrow 15x - 35 = 20 \Rightarrow 15x = 55 \Rightarrow x = \frac{55}{15} = \frac{11}{3}.$$

But it's too much work. Besides, once you find that $x = \frac{11}{3}$, you still have to multiply to get $3x$: $3\left(\frac{11}{3}\right) = 11$, and then subtract to get $3x - 8$: $11 - 8 = \mathbf{3}$.

SOLUTION. The key is to recognize that you don't need to find x. Finding $3x - 7$ is easy (just divide the original equation by 5), and $3x - 8$ is just 1 less:

$$5(3x - 7) = 20 \Rightarrow 3x - 7 = 4 \Rightarrow 3x - 8 = \mathbf{3}, \mathbf{C}.$$

EXAMPLE 16

If $7x + 3y = 17$ and $3x + 7y = 19$, what is the average (arithmetic mean) of x and y?

$\frac{\square}{\square}$

The obvious way to do this is to first find x and y by solving the two equations simultaneously and then to take their average. If you know how to do this, try it now, before reading further. If you worked carefully, you should have found that $x = \frac{31}{20}$ and $y = \frac{41}{20}$, and their average is $\frac{\frac{31}{20} + \frac{41}{20}}{2} = \frac{9}{5}$. Enter 9 as the numerator and 5 as the denominator.

This is not too difficult, but it is quite time-consuming, and questions on the GRE never require you to do that much work. Look for a shortcut. Is there a way to find the average without first finding x and y? Absolutely! Here's the best way to do this.

SOLUTION. Add the two equations:

$$\begin{array}{r} 7x + 3y = 17 \\ +\ 3x + 7y = 19 \\ \hline 10x + 10y = 36 \end{array}$$

Divide each side by 10: $x + y = 3.6$

Calculate the average: $\dfrac{x+y}{2} = \dfrac{3.6}{2} = \mathbf{1.8}$

Since this numeric entry question requires a fraction for the answer, note that $1.8 = 1\frac{8}{10} = \frac{18}{10}$. So enter 18 for the numerator and 10 for the denominator. Remember that you don't have to reduce fractions to lowest terms.

EXAMPLE 17

Benjamin worked from 9:47 A.M. until 12:11 P.M.
Jeremy worked from 9:11 A.M. until 12:47 P.M.

Quantity A	Quantity B
The number of minutes Benjamin worked	The number of minutes Jeremy worked

Do not spend any time calculating how many minutes either of them worked. You only need to know which column is greater, and since Jeremy started earlier and finished later, he clearly worked longer. The answer is **B**.

TACTIC 7 Pay Attention to Units

Often the answer to a question must be in units different from the data given in the question. As you read the question, write on your scratch paper exactly what you are being asked and circle it or put an asterisk next to it. Do they want hours or minutes or seconds, dollars or cents, feet or inches, meters or centimeters? On multiple-choice questions, an answer using the wrong units is almost always one of the choices.

EXAMPLE 18

Driving at 48 miles per hour, how many minutes will it take to drive 32 miles?

Ⓐ $\frac{2}{3}$ Ⓑ $\frac{3}{2}$ Ⓒ 40 Ⓓ 45 Ⓔ 2400

SOLUTION. This is a relatively easy question. Just be attentive. Divide the distance, 32, by the rate, 48: $\frac{32}{48} = \frac{2}{3}$, so it will take $\frac{2}{3}$ of an *hour* to drive 32 miles. Choice A is $\frac{2}{3}$, but that is not the correct answer, because you are asked how many *minutes* it will take. To convert hours to minutes, multiply by 60: it will take $\frac{2}{3}(60) =$ **40** minutes, **C**.

Note that you could have been asked how many *seconds* it would take, in which case the answer would be 40(60) = 2400, Choice E.

EXAMPLE 19

At Nat's Nuts a $2\frac{1}{4}$-pound bag of pistachio nuts costs $6.00. At this rate, what is the cost in cents of a bag weighing 9 ounces?

Ⓐ 1.5 Ⓑ 24 Ⓒ 150 Ⓓ 1350 Ⓔ 2400

SOLUTION. This is a relatively simple ratio, but make sure you get the units right. To do this you need to know that there are 100 cents in a dollar and 16 ounces in a pound.

$$\frac{\text{price}}{\text{weight}} : \frac{6 \text{ dollars}}{2.25 \text{ pounds}} = \frac{600 \text{ cents}}{36 \text{ ounces}} = \frac{x \text{ cents}}{9 \text{ ounces}}.$$

Now cross-multiply and solve: $36x = 5400 \Rightarrow x = \mathbf{150}$, **C**.

TACTIC 8 Systematically Make Lists

When a question asks "how many," often the best strategy is to make a list of all the possibilities. If you do this it is important that you make the list in a *systematic* fashion so that you don't inadvertently leave something out. Usually, this means listing the possibilities in numerical or alphabetical order. Often, shortly after starting the list, you can see a pattern developing and you can figure out how many more entries there will be without writing them all down. Even if the question does not specifically ask "how many," you may need to count something to answer it; in this case, as well, the best plan may be to write out a list.

EXAMPLE 20

A palindrome is a number, such as 93539, that reads the same forward and backward. How many palindromes are there between 100 and 1,000?

SOLUTION. First, write down the numbers that begin and end in 1:

101, 111, 121, 131, 141, 151, 161, 171, 181, 191

Next write the numbers that begin and end in a 2:

202, 212, 222, 232, 242, 252, 262, 272, 282, 292

By now you should see the pattern: there are 10 numbers beginning with 1, 10 beginning with 2, and there will be 10 beginning with 3, 4, ..., 9 for a total of $9 \times 10 = \mathbf{90}$ palindromes.

EXAMPLE 21

The product of three positive integers is 300. If one of them is 5, what is the least possible value of the sum of the other two?

Ⓐ 16 Ⓑ 17 Ⓒ 19 Ⓓ 23 Ⓔ 32

SOLUTION. Since one of the integers is 5, the product of the other two is 60. Systematically, list all possible pairs, (a, b), of positive integers whose product is 60 and check their sums. First let $a = 1$, then 2, and so on.

a	b	$a + b$
1	60	61
2	30	32
3	20	23
4	15	19
5	12	17
6	10	16

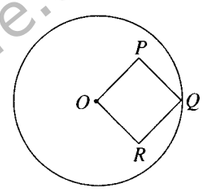
The least possible sum is **16**, **A**.

Practice Exercises

General Math Strategies

1. At Leo's Lumberyard, an 8-foot long wooden pole costs $3.00. At this rate, what is the cost, in cents, of a pole that is 16 inches long?
   Ⓐ 0.5
   Ⓑ 48
   Ⓒ 50
   Ⓓ 64
   Ⓔ 96

2. In the figure below, vertex Q of square $OPQR$ is on a circle with center O. If the area of the square is 8, what is the area of the circle?

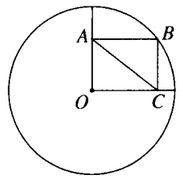


   Ⓐ 8π
   Ⓑ $8\pi\sqrt{2}$
   Ⓒ 16π
   Ⓓ 32π
   Ⓔ 64π

3. In 1999, Diana read 10 English books and 7 French books. In 2000, she read twice as many French books as English books. If 60% of the books that she read during the two years were French, how many books did she read in 2000?
   Ⓐ 16
   Ⓑ 26
   Ⓒ 32
   Ⓓ 39
   Ⓔ 48

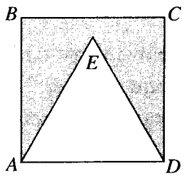
4. In writing all of the integers from 1 to 300, how many times is the digit 1 used?

5. In the figure below, if the radius of circle O is 10, what is the length of diagonal AC of rectangle $OABC$?



   Ⓐ $\sqrt{2}$
   Ⓑ $\sqrt{10}$
   Ⓒ $5\sqrt{2}$
   Ⓓ 10
   Ⓔ $10\sqrt{2}$

6. In the figure below, $ABCD$ is a square and AED is an equilateral triangle. If $AB = 2$, what is the area of the shaded region?

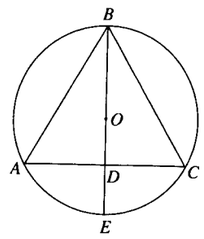


   Ⓐ $\sqrt{3}$
   Ⓑ 2
   Ⓒ 3
   Ⓓ $4 - 2\sqrt{3}$
   Ⓔ $4 - \sqrt{3}$

7. If $5x + 13 = 31$, what is the value of $\sqrt{5x + 31}$?
   Ⓐ $\sqrt{13}$
   Ⓑ $\sqrt{\frac{173}{5}}$
   Ⓒ 7
   Ⓓ 13
   Ⓔ 169

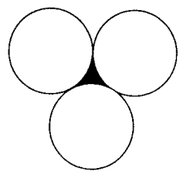
8. If $a + 2b = 14$ and $5a + 4b = 16$, what is the average (arithmetic mean) of a and b?

9. In the figure below, equilateral triangle ABC is inscribed in circle O, whose radius is 4. Altitude BD is extended until it intersects the circle at E. What is the length of DE?

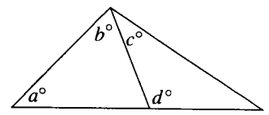


Ⓐ 1
Ⓑ $\sqrt{3}$
Ⓒ 2
Ⓓ $2\sqrt{3}$
Ⓔ $4\sqrt{3}$

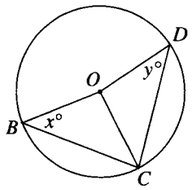
10. In the figure below, three circles of radius 1 are tangent to one another. What is the area of the shaded region between them?



Ⓐ $\frac{\pi}{2} - \sqrt{3}$
Ⓑ 1.5
Ⓒ $\pi - \sqrt{3}$
Ⓓ $\sqrt{3} - \frac{\pi}{2}$
Ⓔ $2 - \frac{\pi}{2}$



	Quantity A	Quantity B
11.	$a + b$	$c + d$



In circle O, $BC > CD$

	Quantity A	Quantity B
12.	x	y

	Quantity A	Quantity B
13.	The number of odd positive factors of 30	The number of even positive factors of 30

Questions 14–15 refer to the following definition.

$\{a, b\}$ represents the remainder when a is divided by b.

	Quantity A	Quantity B
14.	$\{10^3, 3\}$	$\{10^5, 5\}$

c and d are positive integers with $c < d$.

	Quantity A	Quantity B
15.	$\{c, d\}$	$\{d, c\}$

ANSWER KEY

1. **C**	6. **E**	11. **B**
2. **C**	7. **C**	12. **B**
3. **E**	8. **2.5**	13. **C**
4. **160**	9. **C**	14. **A**
5. **D**	10. **D**	15. **A**

ANSWER EXPLANATIONS

Two asterisks (**) indicate an alternative method of solving.

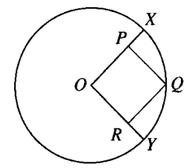
1. **(C)** This is a relatively simple ratio problem, but use TACTIC 7 and make sure you get the units right. To do this you need to know that there are 100 cents in a dollar and 12 inches in a foot.

$$\frac{\text{price}}{\text{weight}} : \frac{3 \text{ dollars}}{8 \text{ feet}} = \frac{300 \text{ cents}}{96 \text{ inches}} = \frac{x \text{ cents}}{16 \text{ inches}}.$$

Now cross-multiply and solve:

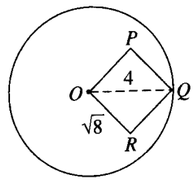
$$96x = 4800 \Rightarrow x = 50.$$

2. **(C)** Use TACTICS 2 and 4. On your scrap paper, extend line segments OP and OR.



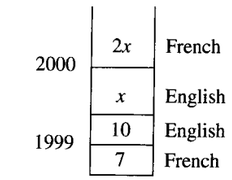
Square $OPQR$, whose area is 8, takes up most of quarter-circle OXY. So the area of the quarter-circle is certainly between 11 and 13. The area of the whole circle is 4 times as great: between 44 and 52. Check the five choices: they are approximately 25, 36, 50, 100, 200. The answer is clearly C.

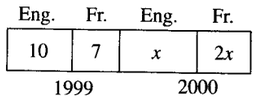
**Another way to use TACTIC 4 is to draw in line segment OQ.



Since the area of the square is 8, each side is $\sqrt{8}$, and diagonal OQ is $\sqrt{8} \times \sqrt{2} = \sqrt{16} = 4$. But OQ is also a radius, so the area of the circle is $\pi(4)^2 = 16\pi$.

3. **(E)** Use TACTIC 1: draw a picture representing a pile of books or a bookshelf.





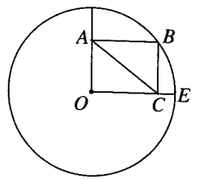
In the two years the number of French books Diana read was $7 + 2x$ and the total number of books was $17 + 3x$. Then 60% or $\frac{3}{5} = \frac{7+2x}{17+3x}$. To solve, cross-multiply:

$$5(7 + 2x) = 3(17 + 3x) \Rightarrow 35 + 10x = 51 + 9x \Rightarrow x = 16.$$

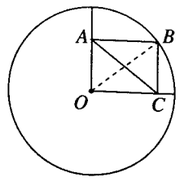
In 2000, Diana read 16 English books and 32 French books, a total of 48 books.

4. **(D)** Use TACTIC 8. Systematically list the numbers that contain the digit 1, writing as many as you need to see the pattern. Between 1 and 99 the digit 1 is used 10 times as the units digit (1, 11, 21, ... , 91) and 10 times as the tens digit (10, 11, 12, ... , 19) for a total of 20 times. From 200 to 299, there are 20 more (the same 20 preceded by a 2). From 100 to 199 there are 20 more plus 100 numbers where the digit 1 is used in the hundreds place. So the total is $20 + 20 + 20 + 100 = 160$.

5. **160** Use TACTIC 2. Trust the diagram: AC, which is clearly longer than OC, is approximately as long as radius OE.



Therefore, AC must be about 10. Check the choices. They are approximately 1.4, 3.1, 7, 10, and 14. The answer must be 10.
**The answer *is* 10. Use TACTIC 4: copy the diagram on your scrap paper and draw in diagonal OB.



Since the two diagonals of a rectangle are equal, and diagonal OB is a radius, $AC = OB = 10$.

6. **(E)** Use TACTIC 5: subtract to find the shaded area. The area of the square is 4.

The area of the equilateral triangle (see Section 11-J) is $\frac{2^2\sqrt{3}}{4} = \frac{4\sqrt{3}}{4} = \sqrt{3}$.

So the area of the shaded region is $4 - \sqrt{3}$.

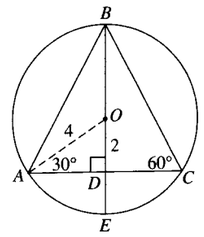
7. **(C)** Use TACTIC 6: don't do more than you have to. In particular, don't solve for x.

$$5x + 13 = 31 \Rightarrow 5x = 18 \Rightarrow 5x + 31 =$$
$$18 + 31 = 49 \Rightarrow \sqrt{5x+31} = \sqrt{49} = 7.$$

8. **2.5** Use TACTIC 6: don't do more than is necessary. We don't need to know the values of a and b, only their average. Adding the two equations, we get

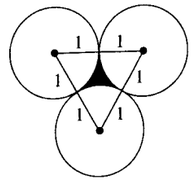
$$6a + 6b = 30 \Rightarrow a + b = 5 \Rightarrow \frac{a+b}{2} = \frac{5}{2} = 2.5.$$

9. **(C)** Use TACTIC 5: to get DE, subtract OD from radius OE, which is 4. Draw AO (TACTIC 4).



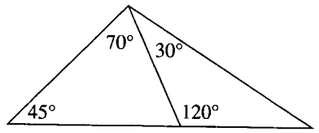
Since $\triangle AOD$ is a 30-60-90 right triangle, OD is 2 (one half of OA). So, $DE = 4 - 2 = 2$.

10. **(D)** Use TACTIC 4 and add some lines: connect the centers of the three circles to form an equilateral triangle whose sides are 2.



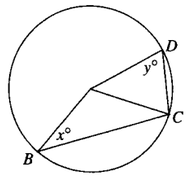
Now use TACTIC 5 and find the shaded area by subtracting the area of the three sectors from the area of the triangle. The area of the triangle is $\frac{2^2\sqrt{3}}{4} = \sqrt{3}$ (see Section 11-J).
Each sector is one sixth of a circle of radius 1. Together they form one half of such a circle, so their total area is $\frac{1}{2}\pi(1)^2 = \frac{\pi}{2}$. Finally, subtract: the shaded area is $\sqrt{3} - \frac{\pi}{2}$.

11. **(B)** If you don't see how to answer this, use TACTIC 2: trust the diagram. Estimate the measure of each angle: for example, $a = 45$, $b = 70$, $c = 30$, and $d = 120$. So $c + d$ (150) is considerably greater than $a + b$ (115). Choose B.



**In fact, d by itself is equal to $a + b$ (an exterior angle of a triangle is equal to the sum of the opposite two interior angles). So $c + d > a + b$.

12. **(B)** From the figure, it appears that x and y are equal, or nearly so. However, the given information states that $BC > CD$, but this is not clear from the diagram. Use TACTIC 3: when you draw the figure on your scrap paper, exaggerate it. Draw it with BC much greater than CD. Now it is clear that y is greater.



13. **(C)** Use TACTIC 8. Systematically list all the factors of 30, either individually or in pairs: 1, 30; 2, 15; 3, 10; 5, 6. Of the 8 factors, 4 are even and 4 are odd.

14. **(A)** Quantity A: When 10^3 (1000) is divided by 3, the quotient is 333 and the remainder is 1. Quantity B: 10^5 is divisible by 5, so the remainder is 0.
Quantity A is greater.

15. **(A)** Quantity A: since $c < d$, the quotient when c is divided by d is 0, and the remainder is c. Quantity B: when d is divided by c the remainder must be less than c.
So Quantity A is greater.

CHAPTER 8

Discrete Quantitative Questions

About 20 of the 40 questions in the two math sections are what the ETS calls discrete quantitative questions. These questions are of three types:

- Multiple-choice questions
- Multiple-answer questions
- Numeric entry questions

Multiple-choice questions are just the standard multiple-choice questions that you are familiar with. Each one has five answer choices, exactly one of which is the correct answer. To get credit for a multiple-choice question you simply click on the oval in front of the one correct answer choice.

Multiple-answer questions are multiple-choice questions with a twist. These questions could have anywhere from 3 to 12 answer choices, any number of which could be correct, from just one to all of them. To alert you to the fact that there may be, and usually is, more than one correct answer, instead of an oval, a square appears in front of each answer choice. To get credit for a multiple-answer question, you must click on the square in front of each correct answer and leave blank the squares in front of each of the incorrect answers.

Numeric entry questions are the only questions on the test for which no answer choices are given. The answer to such a question may be a positive or negative integer, decimal, or fraction. To get credit for a numeric entry question you must use the keyboard to enter your answer into the box on the screen directly below the question. If in answering a question, you use the onscreen calculator and the digital readout is exactly the answer that you want to enter in the box, you can click on the calculator's TRANSFER DISPLAY bar and the readout will automatically appear in the box. Always enter the exact answer unless the question tells you to round your answer, in which case you must round it to the degree of accuracy asked for.

If the answer is to be entered as a fraction, there will be two boxes, and you are to enter the numerator in the upper box and the denominator in the lower box. Any answer equivalent to a correct answer earns full credit. If the correct answer to a question is 2.5, then 2.50 is equally acceptable, unless you were told to give the answer to the nearest tenth. Also, fractions do not have to be reduced: if the correct answer is $\frac{1}{2}$, then you would receive full credit for $\frac{3}{6}$ or $\frac{13}{26}$, or any other fraction

TIP

When you take the GRE, dismiss the instructions for these questions instantly—do not spend even one second reading them—and certainly never accept their offer of clicking on "HELP" to return to them during the test.

TIP

On pages 11–12 you can see a worked out example of each of these three types of questions.

The majority of discrete quantitative questions are of the multiple-choice variety, and all of the tactics discussed in this chapter apply to them. Some of the tactics also apply to multiple-answer questions and numeric entry questions.

The important strategies you will learn in this chapter help you answer many questions on the GRE. However, as invaluable as these tactics are, use them only when you need them. *If you know how to do a problem and are confident that you can do it accurately and reasonably quickly, JUST DO IT!*

As we have done throughout this book, on multiple-choice questions we will continue to label the five answer choices A, B, C, D, and E and to refer to them as such. On multiple-answer questions, the choices will be consecutively labeled A, B, C, etc., using as many letters as there are answer choices. Of course, when you take the GRE, these letters will not appear—there will simply be a blank oval in front of each of the answer choices. When we refer to Choice C—as we do, for example, in TACTIC 1 (below)—we are simply referring to the third answer choice among the five presented.

Testing Tactics

TACTIC 1 Test the Choices, Starting with C

TACTIC 1, often called ***backsolving***, is useful when you are asked to solve for an unknown and you understand what needs to be done to answer the question, but you want to avoid doing the algebra. The idea is simple: test the various choices to see which one is correct.

NOTE: On the GRE the answers to virtually all numerical multiple-choice questions are listed in either increasing or decreasing order. Consequently, C is the middle value, and *in applying TACTIC 1, you should always start with C.* For example, assume that choices A, B, C, D, and E are given in increasing order. Try C. If it works, you've found the answer. If C doesn't work, you should know whether you need to test a larger number or a smaller one, and that permits you to eliminate two more choices. If C is too small, you need a larger number, and so A and B are out; if C is too large, eliminate D and E, which are even larger.

Examples 1 and 2 illustrate the proper use of TACTIC 1.

EXAMPLE 1

If the average (arithmetic mean) of 5, 6, 7, and w is 10, what is the value of w?

Ⓐ 8 Ⓑ 13 Ⓒ 18 Ⓓ 22 Ⓔ 28

SOLUTION.

Use TACTIC 1. Test Choice C: $w = 18$.

- Is the average of 5, 6, 7, and 18 equal to 10?
- No: $\frac{5+6+7+18}{4} = \frac{36}{4} = 9$, which is *too small.*
- Eliminate C, and, since for the average to be 10, w must be *greater* than 18, eliminate A and B, as well.
- Try D: $w = 22$. Is the average of 5, 6, 7, and 22 equal to 10?
- Yes: $\frac{5+6+7+22}{4} = \frac{40}{4} = 10$. The answer is **D**.

Every problem that can be solved using TACTIC 1 can be solved directly, often in less time. So we stress: *if you are confident that you can solve a problem quickly and accurately, just do so.*

Here are two direct methods for solving Example 1, each of which is faster than backsolving. (See Section 11-E on averages.) If you know either method you should use it, and save TACTIC 1 for those problems that you can't easily solve directly.

DIRECT SOLUTION 1. If the average of four numbers is 10, their sum is 40. So, $5 + 6 + 7 + w = 40 \Rightarrow 18 + w = 40 \Rightarrow w =$ **22.**

DIRECT SOLUTION 2. Since 5 is *5 less than* 10, 6 is *4 less than* 10, and 7 is *3 less than* 10, to compensate, w must be $5 + 4 + 3 = 12$ *more than* 10.

So, $w = 10 + 12 =$ **22.**

EXAMPLE 2

Judy is now twice as old as Adam, but 6 years ago, she was 5 times as old as he was. How old is Judy now?

Ⓐ 10 Ⓑ 16 Ⓒ 20 Ⓓ 24 Ⓔ 32

SOLUTION.

Use TACTIC 1: backsolve starting with C. If Judy is now 20, Adam is 10, and 6 years ago, they would have been 14 and 4. Since Judy would have been less than 5 times as old as Adam, eliminate C, D, and E, and try a smaller value. If Judy is now 16, Adam is 8; 6 years ago, they would have been 10 and 2. That's it; 10 *is* 5 times 2. The answer is **B.**

(See Section 11-H on word problems for the correct algebraic solution.)

Some tactics allow you to eliminate a few choices so you can make an educated guess. On those problems where it can be used, TACTIC 1 *always* gets you the right answer. The only reason not to use it on a particular problem is that you can *easily* solve the problem directly.

TIP

Don't start with C if some of the other choices are much easier to work with. If you start with B and it is too small, you may only get to eliminate two choices (A and B), instead of three, but it will save time if plugging in Choice C would be messy.

EXAMPLE 3

If $3x = 2(5 - 2x)$, then $x =$

Ⓐ $-\frac{10}{7}$ Ⓑ 0 Ⓒ $\frac{3}{7}$ Ⓓ 1 Ⓔ $\frac{10}{7}$

SOLUTION.

Since plugging in 0 is so much easier than plugging in $\frac{3}{7}$, start with B: then the left-hand side of the equation is 0 and the right-hand side is 10. The left-hand side is much too small. Eliminate A and B and try something bigger — D, of course; it will be much easier to deal with 1 than with $\frac{3}{7}$ or $\frac{10}{7}$. Now the left-hand side is 3 and the right-hand side is 6. We're closer, but not there. The answer must be **E**. Notice that we got the right answer without ever plugging in one of those unpleasant fractions. Are you uncomfortable choosing E without checking it? Don't be. If you *know* that the answer is greater than 1, and only one choice is greater than 1, that choice has to be right.

Again, we emphasize that, no matter what the choices are, you backsolve *only* if you can't easily do the algebra. Most students would probably do this problem directly:

$$3x = 2(5 - 2x) \Rightarrow 3x = 10 - 4x \Rightarrow 7x = 10 \Rightarrow x = \frac{10}{7}$$

and save backsolving for a harder problem. You have to determine which method is best for you.

TACTIC 2 Replace Variables with Numbers

Mastery of TACTIC 2 is critical for anyone developing good test-taking skills. This tactic can be used whenever the five choices involve the variables in the question. There are three steps:

1. Replace each letter with an easy-to-use number.
2. Solve the problem using those numbers.
3. Evaluate each of the five choices with the numbers you picked to see which choice is equal to the answer you obtained.

Examples 4 and 5 illustrate the proper use of TACTIC 2.

EXAMPLE 4

If a is equal to the sum of b and c, which of the following is equal to the difference of b and c?

Ⓐ $a - b - c$ Ⓑ $a - b + c$ Ⓒ $a - c$ Ⓓ $a - 2c$ Ⓔ $a - b - 2c$

SOLUTION.

- Pick three easy-to-use numbers which satisfy $a = b + c$: for example, $a = 5$, $b = 3$, $c = 2$.
- Then, solve the problem with these numbers: the difference of b and c is $3 - 2 = 1$.
- Finally, check each of the five choices to see which one is equal to 1:

Ⓐ Does $a - b - c = 1$? NO. $5 - 3 - 2 = 0$
Ⓑ Does $a - b + c = 1$? NO. $5 - 3 + 2 = 4$
Ⓒ Does $a - c = 1$? NO. $5 - 2 = 3$
Ⓓ Does $a - 2c = 1$? YES! $5 - 2(2) = 5 - 4 = 1$
Ⓔ Does $a - b - 2c = 1$? NO. $5 - 3 - 2(2) = 2 - 4 = -2$

- The answer is **D**.

EXAMPLE 5

If the sum of five consecutive even integers is t, then, in terms of t, what is the greatest of these integers?

Ⓐ $\frac{t-20}{5}$ Ⓑ $\frac{t-10}{5}$ Ⓒ $\frac{t}{5}$ Ⓓ $\frac{t+10}{5}$ Ⓔ $\frac{t+20}{5}$

SOLUTION.

- Pick five easy-to-use consecutive even integers: say, 2, 4, 6, 8, 10. Then t, their sum, is 30.
- Solve the problem with these numbers: the greatest of these integers is 10.
- When $t = 30$, the five choices are $\frac{10}{5}$, $\frac{20}{5}$, $\frac{30}{5}$, $\frac{40}{5}$, $\frac{50}{5}$.
- Only $\frac{50}{5}$, Choice **E**, is equal to 10.

Of course, Examples 4 and 5 can be solved without using TACTIC 3 *if your algebra skills are good.* Here are the solutions.

SOLUTION 4. $a = b + c \Rightarrow b = a - c \Rightarrow b - c = (a - c) - c = a - 2c$.

SOLUTION 5. Let n, $n + 2$, $n + 4$, $n + 6$, and $n + 8$ be five consecutive even integers, and let t be their sum. Then,

$$t = n + (n + 2) + (n + 4) + (n + 6) + (n + 8) = 5n + 20$$

So, $n = \frac{t-20}{5} \Rightarrow n + 8 = \frac{t-20}{5} + 8 = \frac{t-20}{5} + \frac{40}{5} = \frac{t+20}{5}$.

The important point is that if you can't do the algebra, you can still use TACTIC 2 and *always* get the right answer. Of course, you should use TACTIC 2 even if you can do the algebra, if you think that by using this tactic you will solve the problem faster or will be less likely to make a mistake. This is a good example of what we mean when we say that with the proper use of these tactics, you can correctly answer many questions for which you may not know the correct mathematical solution.

TIP

Replace the letters with numbers that are easy to use, not necessarily ones that make sense. *It is perfectly OK to ignore reality.* A school can have 5 students, apples can cost 10 dollars each, trains can go 5 miles per hour or 1000 miles per hour — it doesn't matter.

Examples 6 and 7 are somewhat different. You are asked to reason through word problems involving only variables. Most students find problems like these mind-boggling. Here, the use of TACTIC 2 is essential. Without it, Example 6 is difficult and Example 7 is nearly impossible. This is not an easy tactic to master, but with practice you will catch on.

EXAMPLE 6

If a school cafeteria needs c cans of soup each week for each student, and if there are s students in the school, for how many weeks will x cans of soup last?

Ⓐ csx Ⓑ $\frac{xs}{c}$ Ⓒ $\frac{s}{cx}$ Ⓓ $\frac{x}{cs}$ Ⓔ $\frac{cx}{s}$

SOLUTION.

- Replace c, s, and x with three easy-to-use numbers. If a school cafeteria needs 2 cans of soup each week for each student, and if there are 5 students in the school, how many weeks will 20 cans of soup last?
- Since the cafeteria needs $2 \times 5 = 10$ cans of soup per week, 20 cans will last 2 weeks.
- Which of the choices equals 2 when $c = 2$, $s = 5$, and $x = 20$?
- $csx = 200$; $\frac{xs}{c} = 50$; $\frac{s}{cx} = \frac{1}{8}$; $\frac{x}{cs} = 2$; and $\frac{cx}{s} = 8$.

The answer is $\frac{x}{cs}$, **D**.

NOTE: You do not need to get the exact value of each choice. As soon as you see that a choice does not equal the value you are looking for, stop—eliminate that choice and move on. For example, in the preceding problem, it is clear that csx is much greater than 2, so eliminate it immediately; you do not need to multiply it out to determine that the value is 200.

CAUTION

In this type of problem it is *not* a good idea to replace any of the variables by 1. Since multiplying and dividing by 1 give the same result, you would not be able to distinguish between $\frac{cx}{s}$ and $\frac{x}{cs}$, both of which are equal to 4 when $c = 1$, $s = 5$, and $x = 20$. It is also not a good idea to use the same number for different variables: $\frac{cx}{s}$ and $\frac{xs}{c}$ are each equal to x when c and s are equal.

EXAMPLE 7

A vendor sells h hot dogs and s sodas. If a hot dog costs twice as much as a soda, and if the vendor takes in a total of d dollars, how many cents does a soda cost?

Ⓐ $\frac{100d}{s+2h}$ Ⓑ $\frac{s+2h}{100d}$ Ⓒ $\frac{d(s+2h)}{100}$ Ⓓ $100d(s+2h)$ Ⓔ $\frac{d}{100(s+2h)}$

SOLUTION.

- Replace h, s, and d with three easy-to-use numbers. Suppose a soda costs 50¢ and a hot dog \$1.00. Then, if he sold 2 sodas and 3 hot dogs, he took in 4 dollars.
- Which of the choices equals 50 when $s = 2$, $h = 3$, and $d = 4$?
- Only $\frac{100d}{s+2h}$ (**A**): $\frac{100(4)}{2+2(3)} = \frac{400}{8} = 50$.

Now, practice TACTIC 3 on the following problems.

EXAMPLE 8

Yann will be x years old y years from now. How old was he z years ago?

Ⓐ $x+y+z$ Ⓑ $x+y-z$ Ⓒ $x-y-z$ Ⓓ $y-x-z$ Ⓔ $z-y-x$

SOLUTION.

Assume that Yann will be 10 in 2 years. How old was he 3 years ago? If he will be 10 in 2 years, he is 8 now and 3 years ago he was 5. Which of the choices equals 5 when $x = 10$, $y = 2$, and $z = 3$? Only $x-y-z$, **C**.

EXAMPLE 9

Stan drove for h hours at a constant rate of r miles per hour. How many miles did he go during the final 20 minutes of his drive?

Ⓐ $20r$ Ⓑ $\frac{hr}{3}$ Ⓒ $3rh$ Ⓓ $\frac{hr}{20}$ Ⓔ $\frac{r}{3}$

SOLUTION.

If Stan drove at 60 miles per hour for 2 hours, how far did he go in the last 20 minutes? Since 20 minutes is $\frac{1}{3}$ of an hour, he went 20 ($\frac{1}{3}$ of 60) miles. Only $\frac{r}{3}$ is 20 when $r = 60$ and $h = 2$. Notice that h is irrelevant. Whether he had been driving for 2 hours or 20 hours, the distance he covered in the last 20 minutes would be the same.

TIP

In problems involving fractions, the best number to use is the least common denominator of all the fractions. In problems involving percents, the easiest number to use is 100. (See Sections 11-B and 11-C.)

TACTIC 3

Choose an Appropriate Number

TACTIC 3 is similar to TACTIC 2, in that we pick convenient numbers. However, here no variable is given in the problem. TACTIC 3 is especially useful in problems involving fractions, ratios, and percents.

EXAMPLE 10

At Madison High School each student studies exactly one foreign language. Three-fifths of the students take Spanish, and one-fourth of the remaining students take German. If all of the others take French, what percent of the students take French?

Ⓐ 10 Ⓑ 15 Ⓒ 20 Ⓓ 25 Ⓔ 30

SOLUTION.

The least common denominator of $\frac{3}{5}$ and $\frac{1}{4}$ is 20, so assume that there are 20 students at Madison High. (Remember the numbers don't have to be realistic.) The number of students taking Spanish is 12 ($\frac{3}{5}$ of 20). Of the remaining 8 students, 2 of them ($\frac{1}{4}$ of 8) take German. The other 6 take French. Finally, 6 is **30%** of 20. The answer is **E**.

EXAMPLE 11

From 1994 to 1995 the sales of a book decreased by 80%. If the sales in 1996 were the same as in 1994, by what percent did they increase from 1995 to 1996?

Ⓐ 80% Ⓑ 100% Ⓒ 120% Ⓓ 400% Ⓔ 500%

SOLUTION.

Since this problem involves percents, assume that 100 copies of the book were sold in 1994 (and 1996). Sales dropped by 80 (80% of 100) to 20 in 1995 and then increased by 80, from 20 back to 100, in 1996. The percent increase was

$$\frac{\text{the actual increase}}{\text{the original amount}} \times 100\% = \frac{80}{20} \times 100\% = \mathbf{400\%}, \mathbf{D}.$$

TACTIC 4

Eliminate Absurd Choices and Guess

When you have no idea how to solve a multiple-choice question, you can always make an educated guess—simply eliminate all the absurd choices and then guess from among the remaining ones.

During the course of a GRE, you will probably find at least a few multiple-choice questions that you don't know how to solve. Since you are not penalized for wrong answers, you are surely going to enter answers for them. But before taking a wild guess, take a moment to look at the answer choices. Often two or three of them are absurd. Eliminate those and then guess one of the others. Occasionally, four of the choices are absurd. When this occurs, your answer is no longer a guess.

What makes a choice absurd? Lots of things. Here are a few. Even if you don't know how to solve a problem you may realize that

- the answer must be positive, but some of the choices are negative;
- the answer must be even, but some of the choices are odd;
- the answer must be less than 100, but some choices exceed 100;
- a ratio must be less than 1, but some choices are greater than 1.

Let's look at several examples. In a few of them the information given is intentionally insufficient to solve the problem; but you will still be able to determine that some of the answers are absurd. In each case the "solution" will indicate which choices you should have eliminated. At that point you would simply guess. Remember, on the GRE when you guess, don't agonize. Just guess and move on.

EXAMPLE 12

A region inside a semicircle of radius r is shaded and you are asked for its area.

Ⓐ $\frac{1}{4}\pi r^2$ Ⓑ $\frac{1}{3}\pi r^2$ Ⓒ $\frac{1}{2}\pi r^2$ Ⓓ $\frac{2}{3}\pi r^2$ Ⓔ πr^2

SOLUTION.

You may have no idea how to find the area of the shaded region, but you should know that since the area of a circle is πr^2, the area of a semicircle is $\frac{1}{2}\pi r^2$. Therefore, the area of the shaded region must be *less* than $\frac{1}{2}\pi r^2$, so eliminate C, D, and E. On an actual GRE problem, you may be able to make an educated guess between A and B. If so, terrific; if not, just choose one or the other.

EXAMPLE 13

The average (arithmetic mean) of 5, 10, 15, and z is 20. What is z?

Ⓐ 0 Ⓑ 20 Ⓒ 25 Ⓓ 45 Ⓔ 50

SOLUTION.
If the average of four numbers is 20, and three of them are less than 20, the other one must be greater than 20. Eliminate A and B and guess. If you further realize that since 5 and 10 are a *lot less* than 20, z will probably be a *lot more* than 20; eliminate C, as well.

EXAMPLE 14

If 25% of 260 equals 6.5% of a, what is a?

Ⓐ 10 Ⓑ 65 Ⓒ 100 Ⓓ 130 Ⓔ 1000

SOLUTION.
Since 6.5% of a equals 25% of 260, which is surely greater than 6.5% of 260, a must be greater than 260. Eliminate A, B, C, and D. The answer *must* be **E**!

Example 14 illustrates an important point. *Even if you know how to solve a problem,* if you immediately see that four of the five choices are absurd, just pick the fifth choice and move on.

EXAMPLE 15

A jackpot of $39,000 is to be divided in some ratio among three people. What is the value of the largest share?

Ⓐ $23,400 Ⓑ $19,500 Ⓒ $11,700 Ⓓ $7800 Ⓔ $3900

SOLUTION.
If the prize were divided equally, each of the three shares would be worth $13,000. If it is divided unequally, the largest share is surely worth *more than* $13,000. Eliminate C, D, and E. In an actual question, you would be told what the ratio is, and that might enable you to eliminate A or B. If not, you just guess.

EXAMPLE 16

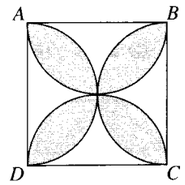
In a certain club, the ratio of the number of boys to girls is 5:3. What percent of the members of the club are girls?

Ⓐ 37.5% Ⓑ 50% Ⓒ 60% Ⓓ 62.5% Ⓔ 80%

SOLUTION.
Since there are 5 boys for every 3 girls, there are fewer girls than boys. Therefore, *fewer than half* (50%) of the members are girls. Eliminate B, C, D, and E. The answer is **A**.

EXAMPLE 17

In the figure below, four semicircles are drawn, each one centered at the midpoint of one of the sides of square $ABCD$. Each of the four shaded "petals" is the intersection of two of the semicircles. If $AB = 4$, what is the total area of the shaded region?



Ⓐ 8π Ⓑ $32 - 8\pi$ Ⓒ $16 - 8\pi$ Ⓓ $8\pi - 32$ Ⓔ $8\pi - 16$

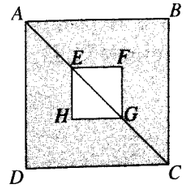
SOLUTION.
- Since $AB = 4$, the area of the square is 16, and so, obviously, the area of the shaded region must be much less.
- Check each choice. Since π is slightly more than 3 ($\pi \approx 3.14$), 8π is somewhat greater than 24, approximately 25.
- (A) $8\pi \approx 25$. More than the area of the whole square: way too big.
- (B) $32 - 8\pi \approx 32 - 25 = 7$.
- (C) $16 - 8\pi$ is negative.
- (D) $8\pi - 32$ is also negative.
- (E) $8\pi - 16 \approx 25 - 16 = 9$.

NOTE: Three of the choices are absurd: A is more than the area of the entire square and C and D are negative; they can be eliminated immediately. The answer must be B or E. If you think the shaded area takes up less than half of the square, guess B; if you think it takes up more than half of the square, guess E. (The answer is **E**).

Now use TACTIC 4 on each of the following problems. Even if you know how to solve them, don't. Practice this technique and see how many choices you can eliminate *without* actually solving.

EXAMPLE 18

In the figure at the right, diagonal EG of square $EFGH$ is $\frac{1}{2}$ of diagonal AC of the square $ABCD$. What is the ratio of the area of the shaded region to the area of $ABCD$?



Ⓐ $\sqrt{2}:1$ Ⓑ 3:4 Ⓒ $\sqrt{2}:2$ Ⓓ 1:2 Ⓔ $1:2\sqrt{2}$

SOLUTION.
Obviously, the shaded region is smaller than square *ABCD*, so the ratio must be less than 1. Eliminate A. Also, from the diagram, it is clear that the shaded region is more than half of square *ABCD*, so the ratio is greater than 0.5. Eliminate D and E. Since 3:4 = .75 and $\sqrt{2}$:2 ≈ .71, B and C are too close to tell which is correct just by looking; so guess. The answer is **B**.

EXAMPLE 3

Shari receives a commission of 25¢ for every $20.00 worth of merchandise she sells. What percent is her commission?

Ⓐ $1\frac{1}{4}\%$ Ⓑ $2\frac{1}{2}\%$ Ⓒ 5% Ⓓ 25% Ⓔ 125%

SOLUTION.
Clearly, a commission of 25¢ on $20 is quite small. Eliminate D and E and guess one of the small percents. If you realize that 1% of $20 is 20¢, then you know the answer is a little more than 1%, and you should guess A (maybe B, but definitely not C). The answer is **A**.

EXAMPLE 4

From 1980 to 1990, Lior's weight increased by 25%. If his weight was *k* kilograms in 1990, what was it in 1980?

Ⓐ 1.75*k* Ⓑ 1.25*k* Ⓒ 1.20*k* Ⓓ .80*k* Ⓔ .75*k*

SOLUTION.
Since Lior's weight increased, his weight in 1980 was *less than k*. Eliminate A, B, and C and guess. The answer is **D**.

EXAMPLE 5

The average of 10 numbers is –10. If the sum of 6 of them is 100, what is the average of the other 4?

Ⓐ –100 Ⓑ –50 Ⓒ 0 Ⓓ 50 Ⓔ 100

SOLUTION.
Since the average of all 10 numbers is negative, so is their sum. But the sum of the first 6 is positive, so the sum (and the average) of the others must be negative. Eliminate C, D, and E. **B** is correct.

Practice Exercises

Discrete Quantitative Questions

1. Evan has 4 times as many books as David and 5 times as many as Jason. If Jason has more than 40 books, what is the least number of books that Evan could have?
   Ⓐ 200
   Ⓑ 205
   Ⓒ 210
   Ⓓ 220
   Ⓔ 240

2. Judy plans to visit the National Gallery once each month in 2012 except in July and August when she plans to go three times each. A single admission costs $3.50, a pass valid for unlimited visits in any 3-month period can be purchased for $18, and an annual pass costs $60.00. What is the least amount, in dollars, that Judy can spend for her intended number of visits?
   ☐ dollars

3. Alison is now three times as old as Jeremy, but 5 years ago, she was 5 times as old as he was. How old is Alison now?
   Ⓐ 10
   Ⓑ 12
   Ⓒ 24
   Ⓓ 30
   Ⓔ 36

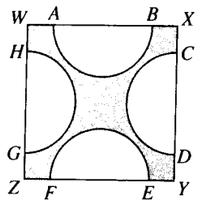
4. What is the largest prime factor of 255?
   Ⓐ 5
   Ⓑ 15
   Ⓒ 17
   Ⓓ 51
   Ⓔ 255

5. If *c* is the product of *a* and *b*, which of the following is the quotient of *a* and *b*?
   Ⓐ $\frac{b^2}{c}$
   Ⓑ $\frac{c}{b^2}$
   Ⓒ $\frac{b}{c^2}$
   Ⓓ bc^2
   Ⓔ b^2c

6. If *w* widgets cost *c* cents, how many widgets can you get for *d* dollars?
   Ⓐ $\frac{100dw}{c}$
   Ⓑ $\frac{dw}{100c}$
   Ⓒ $100cdw$
   Ⓓ $\frac{dw}{c}$
   Ⓔ cdw

7. If 120% of *a* is equal to 80% of *b*, which of the following is equal to *a* + *b*?
   Ⓐ 1.5*a*
   Ⓑ 2*a*
   Ⓒ 2.5*a*
   Ⓓ 3*a*
   Ⓔ 5*a*

8. In the figure below, *WXYZ* is a square whose sides are 12. *AB*, *CD*, *EF*, and *GH* are each 8, and are the diameters of the four semicircles. What is the area of the shaded region?



Ⓐ $144 - 128\pi$
Ⓑ $144 - 64\pi$
Ⓒ $144 - 32\pi$
Ⓓ $144 - 16\pi$
Ⓔ 16π

9. If x and y are integers such that $x^3 = y^2$, which of the following could not be the value of y? Indicate *all* such values.

[A] -1
[B] 1
[C] 8
[D] 12
[E] 16
[F] 27

10. What is a divided by a% of a?

Ⓐ $\frac{a}{100}$

Ⓑ $\frac{100}{a}$

Ⓒ $\frac{a^2}{100}$

Ⓓ $\frac{100}{a^2}$

Ⓔ $100a$

11. If an object is moving at a speed of 36 kilometers per hour, how many meters does it travel in one second?

[ ] meters

12. On a certain French-American committee, $\frac{2}{3}$ of the members are men, and $\frac{3}{8}$ of the men are Americans. If $\frac{3}{5}$ of the committee members are French, what fraction of the members are American women?

$\frac{\square}{\square}$

13. For what value of x is $8^{2x-4} = 16^x$?

Ⓐ 2
Ⓑ 3
Ⓒ 4
Ⓓ 6
Ⓔ 8

14. If $12a + 3b = 1$ and $7b - 2a = 9$, what is the average (arithmetic mean) of a and b?

Ⓐ 0.1
Ⓑ 0.5
Ⓒ 1
Ⓓ 2.5
Ⓔ 5

15. If x% of y is 10, what is y?

Ⓐ $\frac{10}{x}$

Ⓑ $\frac{100}{x}$

Ⓒ $\frac{1000}{x}$

Ⓓ $\frac{x}{100}$

Ⓔ $\frac{x}{10}$

ANSWER KEY

1. **D**	6. **A**	11. **10**	14. **B**
2. **49.50**	7. **C**	12. $\frac{\mathbf{3}}{\mathbf{20}}$	15. **C**
3. **D**	8. **C**	13. **D**	
4. **C**	9. **D, E**		
5. **B**	10. **B**		

ANSWER EXPLANATIONS

Two asterisks (**) indicate an alternative method of solving.

1. **(D)** Test the answer choices starting with the smallest value. If Evan had 200 books, Jason would have 40. But Jason has more than 40, so 200 is too small. Trying 205 and 210, we see that neither is a multiple of 4, so David wouldn't have a whole number of books. Finally, 220 works. (So does 240, but we shouldn't even test it since we want the least value.)

**Since Jason has at least 41 books, Evan has at least $41 \times 5 = 205$. But Evan's total must be a multiple of 4 and 5, hence of 20. The smallest multiple of 20 greater than 205 is 220.

2. **49.50** Judy intends to go to the Gallery 16 times during the year. Buying a single admission each time would cost $16 \times \$3.50 = \$56$, which is less than the annual pass. If she bought a 3-month pass for June, July, and August, she would pay \$18 plus \$31.50 for 9 single admissions ($9 \times \$3.50$), for a total expense of \$49.50, which is the least expensive option.

3. **(D)** Use TACTIC 1: backsolve starting with C. If Alison is now 24, Jeremy is 8, and 5 years ago, they would have been 19 and 3, which is more than 5 times as much. Eliminate A, B, and C, and try a bigger value. If Alison is now 30, Jeremy is 10, and 5 years ago, they would have been 25 and 5. That's it; 25 is 5 times 5.

**If Jeremy is now x, Alison is $3x$, and 5 years ago they were $x - 5$ and $3x - 5$, respectively. Now, solve:

$3x - 5 = 5(x - 5) \Rightarrow 3x - 5 = 5x - 25 \Rightarrow$
$2x = 20 \Rightarrow x = 10 \Rightarrow 3x = 30.$

4. **(C)** Test the choices starting with C: 255 *is* divisible by 17 ($255 = 17 \times 15$), so this is a possible answer. Does 255 have a larger prime factor? Neither Choice D nor E is prime, so the answer must be Choice C.

5. **(B)** Use TACTIC 2. Pick simple values for a, b, and c. Let $a = 3$, $b = 2$, and $c = 6$. Then $a \div b = 3/2$. Without these values of a, b, and c, only B is equal to 3/2.

** $c = ab \Rightarrow a = \frac{c}{b} \Rightarrow a \div b = \frac{c}{b} \div b = \frac{c}{b} \cdot \frac{1}{b} = \frac{c}{b^2}$.

6. **(A)** Use TACTIC 2. If 2 widgets cost 10 cents, then widgets cost 5 cents each, and for 3 dollars, you can get 60. Which of the choices equals 60 when $w = 2$, $c = 10$, and $d = 3$? Only A.

** $\frac{\text{widgets}}{\text{cents}} = \frac{w}{c} = \frac{x}{100d} \Rightarrow x = \frac{100dw}{c}$.

7. **(C)** Since 120% of 80 = 80% of 120, let $a = 80$ and $b = 120$. Then $a + b = 200$, and $200 \div 80 = 2.5$.

8. **(C)** If you don't know how to solve this, you must use TACTIC 4 and guess after eliminating the absurd choices. Which choices are absurd? Certainly, A and B, both of which are negative. Also, since Choice D is about 94, which is much more than half the area of the square, it is much too big. Guess between Choice C (about 43) and Choice E (about 50). If you remember that the way to find shaded areas is to subtract, guess C.

   **The area of the square is $12^2 = 144$. The area of each semicircle is 8π, one-half the area of a circle of radius 4. So together the areas of the semicircles is 32π.

9. **(D)(E)** Test each choice until you find all the correct answers.

   (A) Could $y = -1$? Is there an integer x such that $x^3 = (-1)^2 = 1$? Yes, $x = 1$.
   (B) Similarly, if $y = 1$, $x = 1$.
   (C) Could $y = 8$? Is there an integer x such that $x^3 = (8)^2 = 64$? Yes, $x = 4$.
   (D) Could $y = 12$? Is there an integer such that $x^3 = 12^2 = 144$? No, $5^3 = 125$, which is too small, and $6^3 = 216$, which is too big.
   (E) Could $y = 16$? Is there an integer x such that $x^3 = 16^2 = 256$? No, $6^3 = 216$, which is too small; and $7^3 = 343$, which is too big.
   (F) Could $y = 27$? Is there an integer x such that $x^3 = 27^2 = 729$? Yes, $9^3 = 729$.
   The answer is D and E.

10. **(B)** $a \div (a\% \text{ of } a) = a \div \left(\frac{a}{100} \times a\right) = a \div \left(\frac{a^2}{100}\right) = a \times \frac{100}{a^2} = \frac{100}{a}$.

    **Use TACTICS 2 and 3: replace a by a number, and use 100 since the problem involves percents. $100 \div (100\% \text{ of } 100) = 100 \div 100 = 1$. Test each choice; which ones equal 1 when $a = 100$. Both A and B: $\frac{100}{100} = 1$. Eliminate Choices C, D, and E, and test A and B with another value for a. $50 \div (50\% \text{ of } 50) = 50 \div (25) = 2$. Now, only B works $\left(\frac{100}{50} = 2, \text{ whereas } \frac{50}{100} = \frac{1}{2}\right)$.

11. **10** Set up a ratio:

    $$\frac{\text{distance}}{\text{time}} = \frac{36 \text{ kilometers}}{1 \text{ hour}} = \frac{36{,}000 \text{ meters}}{60 \text{ minutes}} = \frac{36{,}000 \text{ meters}}{3600 \text{ seconds}} = 10 \text{ meters/second.}$$

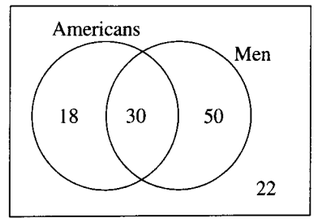
    **Use TACTIC 1: Test choices starting with C:

    100 meters/second = 6000 meters/minute = 360,000 meters/hour = 360 kilometers/hour.

    Not only is that too big, it is too big by a factor of 10. The answer is 10.

12. **$\frac{3}{20}$** Use TACTIC 3. The LCM of all the denominators is 120, so assume that the committee has 120 members. Then there are $\frac{2}{3} \times 120 = 80$ men and 40 women. Of the 80 men 30 $\left(\frac{3}{8} \times 80\right)$ are American. Since there are 72 $\left(\frac{3}{5} \times 120\right)$ French members, there are $120 - 72 = 48$ Americans, of whom 30 are men, so the other 18 are women. Finally, the fraction of American women is $\frac{18}{120} = \frac{3}{20}$.

    This is illustrated in the Venn diagram below.



13. **(D)** Use the laws of exponents to simplify the equation, and then solve it: $8^{2x-4} = 16^x \Rightarrow (2^3)^{2x-4} = (2^4)^x \Rightarrow 3(2x-4) = 4x \Rightarrow 6x - 12 = 4x \Rightarrow 2x = 12 \Rightarrow x = 6$.

14. **(B)** Add the two equations:

    $10a + 10b = 10 \Rightarrow a + b = 1 \Rightarrow \frac{a+b}{2} = \frac{1}{2}$.

    Do not waste time solving for a and b.

15. **(C)** Pick easy-to-use numbers. Since 100% of 10 is 10, let $x = 100$ and $y = 10$. When $x = 100$, Choices C and E are each 10. Eliminate Choices A, B, and D, and try some other numbers: 50% of 20 is 10. Of Choices C and E, only C = 20 when $x = 50$.

CHAPTER 9

Quantitative Comparison Questions

About 15 of the 40 questions on the two quantitative sections of the GRE are quantitative comparisons. Unless you took the SAT before 2005, it is very likely that you have never seen questions of this type and certainly never learned the correct strategies for answering them. Don't worry. In this chapter you will learn all of the necessary tactics. If you master them, you will quickly realize that quantitative comparisons are the easiest mathematics questions on the GRE and will wish that there were more than 15 of them.

Before the first quantitative comparison question appears on the screen, you will see these instructions.

Directions: In the following question, there are two quantities, labeled Quantity A and Quantity B. You are to compare those quantities, taking into consideration any additional information given and decide which of the following statements is true:

Quantity A is greater;
Quantity B is greater;
The two quantities are equal; or
It is impossible to determine which quantity is greater.

Note: The given information, if any, is centered above the two quantities. If a symbol appears more than once, it represents the same thing each time.

Before learning the different strategies for solving this type of question, let's clarify these instructions. In quantitative comparison questions there are two quantities, and it is your job to compare them. The correct answer to a quantitative comparison question is one of the four statements listed in the directions above. Of course, on the computer screen those choices will not be listed as A, B, C, and D. Rather, you will see an oval in front of each statement, and you will click on the oval in front of the statement you believe is true.

You should click on the oval in front of	if
Quantity A is greater.	Quantity A is greater *all the time, no matter what.*
Quantity B is greater.	Quantity B is greater *all the time, no matter what.*
The two quantities are equal.	The two quantities are equal *all the time, no matter what.*
It is impossible to determine which quantity is greater.	*The answer is not one of the first three choices.*

This means, for example, that *if you can find a single instance* when Quantity A is greater than Quantity B, then you can immediately eliminate two choices: the answer cannot be "Quantity B is greater," and the answer cannot be "The two quantities are equal." In order for the answer to be "Quantity B is greater," Quantity B would have to be greater *all the time*; but you know of one instance when it isn't. Similarly, since the quantities are not equal *all the time*, the answer can't be "The two quantities are equal." The correct answer, therefore, is either "Quantity A is greater" or "It is impossible to determine which quantity is greater." If it turns out that Quantity A *is* greater all the time, then that is the answer; if, however, you can find a single instance where Quantity A is not greater, the answer is "It is impossible to determine which quantity is greater."

By applying the tactics that you will learn in this chapter, you will probably be able to determine which of the choices is correct; if, however, after eliminating two of the choices, you still cannot determine which answer is correct, quickly guess between the two remaining choices and move on.

Before learning the most important tactics for handling quantitative comparison questions, let's look at two examples to illustrate the preceding instructions.

TIP

Right now, memorize the instructions for answering quantitative comparison questions. *When you take the GRE, dismiss the instructions for these questions immediately—do not spend even one second reading the directions (or looking at a sample problem).*

EXAMPLE 1

$1 < x < 3$

Quantity A	Quantity B
x^2	$2x$

- ○ Quantity A is greater.
- ○ Quantity B is greater.
- ○ The two quantities are equal.
- ○ It is impossible to determine which quantity is greater.

SOLUTION.

Throughout, x represents the same thing — a number between 1 and 3. If x is 2, then x^2 and $2x$ are each 4, and *in this case* the two quantities are equal. We can, therefore, eliminate the first two choices: neither Quantity A nor Quantity B is greater *all the time.* However, in order for the correct answer to be "The two quantities are equal," the quantities would have to be equal *all the time.* Are they? Note that although 2 is the only *integer* between 1 and 3, it is not the only *number* between 2 and 3: x could be 1.1 or 2.5 or any of infinitely many other numbers. And in those cases the quantities are not equal (for example, $2.5^2 = 6.25$, whereas $2(2.5) = 5$). The quantities are *not* always equal, and so the correct answer is the fourth choice: It is impossible to determine which quantity is greater.

EXAMPLE 2

p and q are primes

$p + q = 12$

Quantity A	Quantity B
p	8

- ○ Quantity A is greater.
- ○ Quantity B is greater.
- ○ The two quantities are equal.
- ○ It is impossible to determine which quantity is greater.

SOLUTION.

Since 5 and 7 are the only primes whose sum is 12, p could be 5 or 7. In either case, p is less than 8, and so Quantity B is greater, *all the time.* Note that although $1 + 11 = 12$, p cannot be 11, because 1 is not a prime [See Section 11-A].

NOTE: To simplify the discussion, throughout the rest of this chapter, in the explanations of the answers to all sample questions and in the Model Tests, the four answer choices will be referred to as A, B, C, and D, respectively. For example, we will write

The correct answer is **B.**

rather than

The correct answer is: Quantity B is greater.

Testing Tactics

TACTIC 1

Replace Variables with Numbers

Many problems that are hard to analyze because they contain variables become easy to solve when the variables are replaced by simple numbers.

TACTIC 1 is the most important tactic in this chapter. Using it properly will earn you more points on the quantitative comparison questions of the GRE than you can gain by applying any of the others. *Be sure to master it!*

Most quantitative comparison questions contain variables. When those variables are replaced by simple numbers such as 0 or 1, the quantities become much easier to compare.

The reason that TACTIC 1 is so important is that it *guarantees* that on any quantitative comparison question that involves variables, you will be able to immediately eliminate two of the four choices, and very often a third choice as well, leaving you with at least a 50% chance of guessing correctly, and often a certainty. Try the following example, and then read the explanation very carefully.

EXAMPLE 3

$a < b < c < d$

Quantity A	Quantity B
ab	cd

SOLUTION.

- Replace a, b, c, and d with easy-to-use numbers which satisfy the condition $a < b < c < d$: for example, $a = 1$, $b = 3$, $c = 6$, $d = 10$. [See the guidelines that follow to learn why 1, 2, 3, 4 is not a good choice.]
- Evaluate the two quantities: $ab = (1)(3) = 3$, and $cd = (6)(10) = 60$.
- So *in this case*, Quantity B is greater.
- Does that mean that B is the correct answer? Not necessarily. Quantity B *is* greater this time, but will it be greater ***every single time, no matter what***?
- What it does mean is that neither A nor C could possibly be the correct answer: Quantity A can't be greater ***every single time, no matter what*** because it isn't greater *this* time; and the quantities aren't equal ***every single time, no matter what*** because they aren't equal *this* time.

So in the few seconds that it took you to plug in 1, 3, 6, and 10 for a, b, c, and d, you were able to eliminate two of the four choices. You now know that the correct answer is either B or D, and if you could do nothing else, you would now guess with a 50% chance of being correct.

But, of course, *you will do something else.* You will try some other numbers. But *which* numbers? Since the first numbers you chose were positive, try some negative numbers this time.

- Let $a = -5$, $b = -3$, $c = -2$, and $d = -1$.
- Evaluate: $ab = (-5)(-3) = 15$ and $cd = (-2)(-1) = 2$.
- So *in this case*, Quantity A is greater.
- Quantity B is *not* greater all the time. B is *not* the correct answer.
- The correct answer is **D**: It is impossible to determine which quantity is greater.

NOTES:

1. If for your second substitution you had chosen 3, 7, 8, 10 or 2, 10, 20, 35 or *any* four positive numbers, Quantity B would have been bigger. No matter how many substitutions you made, Quantity B would have been bigger each time, and you would have incorrectly concluded that B was the answer. In fact, if the given condition had been $0 < a < b < c < d$, then B *would have been* the correct answer.
2. Therefore, knowing which numbers to plug in when you use TACTIC 1 is critical. As long as you comply with the conditions given in the question, you have complete freedom in choosing the numbers. Some choices, however, are much better than others.

Here are some guidelines for deciding which numbers to use when applying TACTIC 1.

1. **The very best numbers to use first are: 1, 0, and –1.**
2. **Often, fractions between 0 and 1 are useful.**
3. **Occasionally, "large" numbers such as 10 or 100 can be used.**
4. **If there is more than one letter, it is permissible to replace each with the same number.**
5. **Do not impose any conditions not specifically stated.** In particular, do not assume that variables must be integers. For example, 3 is not the only number that satisfies $2 < x < 4$ (2.1, 3.95, and π all work). The expression $a < b < c < d$ does not mean that a, b, c, d are *integers*, let alone *consecutive* integers (which is why we didn't choose 1, 2, 3, and 4 in Example 3), nor does it mean that any or all of them are *positive.*

When you replace the variables in a quantitative comparison question with numbers, remember:

If the value of Quantity A is ever greater:	eliminate B and C — the answer must be A or D.
If the value of Quantity B is ever greater:	eliminate A and C — the answer must be B or D.
If the two quantities are ever equal:	eliminate A and B — the answer must be C or D.

You have learned that, no matter how hard a quantitative comparison is, as soon as you replace the variables, two choices can *immediately* be eliminated; and if you can't decide between the other two, just guess. This guarantees that in addition to correctly answering all the questions that you know how to solve, you will be able to answer correctly at least half, and probably many more, of the questions that you don't know how to do.

Practice applying TACTIC 1 on these examples.

EXAMPLE 4

$m > 0$ and $m \neq 1$

Quantity A	Quantity B
m^2	m^3

SOLUTION.
Use TACTIC 1. Replace m with numbers satisfying $m > 0$ and $m \neq 1$.

	Quantity A	Quantity B	Compare	Eliminate
Let $m = 2$	$2^2 = 4$	$2^3 = 8$	B is greater	A and C
Let $m = \frac{1}{2}$	$\left(\frac{1}{2}\right)^2 = \frac{1}{4}$	$\left(\frac{1}{2}\right)^3 = \frac{1}{8}$	A is greater	B

The answer is **D**.

EXAMPLE 5

Quantity A	Quantity B
$13y$	$15y$

SOLUTION.
Use TACTIC 1. There are no restrictions on y, so use the best numbers: 1, 0, –1.

	Quantity A	Quantity B	Compare	Eliminate
Let $y = 1$	13(1) = 13	15(1) =15	B is greater	A and C
Let $y = 0$	13(0) = 0	15(0) = 0	They're equal	B

The answer is **D**.

EXAMPLE 6

Quantity A	Quantity B
$w + 11$	$w - 11$

SOLUTION.
Use TACTIC 1. There are no restrictions on w, so use the best numbers: 1, 0, –1.

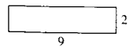
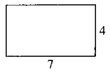
	Quantity A	Quantity B	Compare	Eliminate
Let $w = 1$	1 + 11 = 12	1 – 11 = –10	A is greater	B and C
Let $w = 0$	0 + 11 = 11	0 – 11 = –11	A is greater	
Let $w = -1$	–1 + 11 = 10	–1 – 11 = –12	A is greater	

Guess **A.** We let w be a positive number, a negative number, and 0. Each time, Quantity A was greater. That's not proof, but it justifies an educated guess. [The answer *is* A. Clearly, $11 > -11$, and if we add w to each side, we get: $w + 11 > w - 11$.]

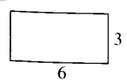
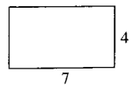
EXAMPLE 7

Quantity A	Quantity B
The perimeter of a rectangle whose area is 18	The perimeter of a rectangle whose area is 28

SOLUTION.
What's this question doing here? How can we use TACTIC 1? Where are the variables that we're supposed to replace? Well, each quantity is the perimeter of a rectangle, and the variables are the lengths and widths of these rectangles.

Quantity A	Quantity B	Compare	Eliminate
Choose a rectangle whose area is 18: 	Choose a rectangle whose area is 28: 		
The perimeter here is 9 + 2 + 9 + 2 = **22**	The perimeter here is 7 + 4 + 7 + 4 = **22**	Quantities A and B are equal	A and B

Keep Quantity B, but take a different rectangle of area 18 when evaluating Quantity A:

			
Perimeter = 3 + 6 + 3 + 6 = **18**	Perimeter = **22**	B is greater	C

The answer is **D**.

EXAMPLE 8

$a = \frac{2}{3}t \quad b = \frac{5}{6}t \quad c = \frac{3}{5}b$

Quantity A	Quantity B
$3a$	$4c$

SOLUTION.
Use TACTIC 1. First, try the easiest number: let $t = 0$. Then a, b, and c are each 0, and *in this case*, the quantities are equal — they're both 0. Eliminate A and B. Now, try another number for t. The obvious choice is 1, but then a, b, and c will all be fractions. To avoid this, let $t = 6$. Then, $a = \frac{2}{3}(6) = 4$, $b = \frac{5}{6}(6) = 5$, and $c = \frac{3}{5}(5) = 3$. This time, $3a = 3(4) = \mathbf{12}$ and $4b = 4(3) = \mathbf{12}$. *Again, the two quantities are equal.* Choose **C**.

NOTE: You should consider answering this question directly (i.e., without plugging in numbers), *only if you are very comfortable with both fractions and elementary algebra.* Here's the solution:

$$c = \frac{3}{5}b = \frac{3}{5}\left(\frac{5}{6}t\right) = \frac{1}{2}t$$

Therefore, $2c = t$, and $4c = 2t$. Since $a = \frac{2}{3}t$, $3a = 2t$. So, $4c = 3a$. The answer is **C**.

TACTIC 2 Choose an Appropriate Number

This is just like TACTIC 1. We are replacing a variable with a number, but the variable isn't mentioned in the problem.

EXAMPLE 9

Every band member is either 15, 16, or 17 years old.
One third of the band members are 16, and
twice as many band members are 16 as 15.

Quantity A	Quantity B
The number of 17-year-old band members	The total number of 15- and 16-year-old band members

If the first sentence of Example 9 had been "There are n students in the school band, all of whom are 15, 16, or 17 years old," the problem would have been identical to this one. Using TACTIC 1, you could have replaced n with an easy-to-use number, such as 6, and solved: $\frac{1}{3}(6) = 2$ are 16 years old; 1 is 15, and the remaining 3 are 17. The answer is **C**.

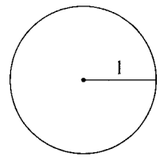
The point of TACTIC 2 is that you can plug in numbers even if there are no variables. As discussed in TACTIC 3, Chapter 8, this is especially useful on problems involving percents, in which case 100 is a good number, and problems involving fractions, in which case the LCD of the fractions is a good choice. However, the use of TACTIC 2 is not limited to these situations. Try using TACTIC 2 on the following three problems.

EXAMPLE 10

The perimeter of a square and the
circumference of a circle are equal.

Quantity A	Quantity B
The area of the circle	The area of the square

SOLUTION.
First use TACTIC 1, Chapter 7: draw a diagram.



$C = 2\pi(1) = 2\pi$
$A = \pi(1)^2 = \pi \approx 3.14$

Then use TACTIC 2: choose an easy-to-use number. Let the radius of the circle be 1. Then its area is π. Let s be the side of the square:

$P = 4s$
$A = s^2$

$4s = 2\pi \approx 6 \Rightarrow s \approx 1.5 \Rightarrow$
area of the square $\approx (1.5)^2 = 2.25$

The answer is **A**.

EXAMPLE 11

Jen, Ken, and Len divided a cash prize.

Jen took 50% of the money and spent $\frac{3}{5}$ of what she took.

Ken took 40% of the money and spent $\frac{3}{4}$ of what he took.

Quantity A	Quantity B
The amount that Jen spent	The amount that Ken spent

SOLUTION.
Use TACTIC 2. Assume the prize was \$100. Then Jen took \$50 and spent $\frac{3}{5}(\$50) = \$30$. Ken took \$40 and spent $\frac{3}{4}(\$40) = \$30$. The answer is **C**.

EXAMPLE 12

Eliane types twice as fast as Delphine.
Delphine charges 50% more per page than Eliane.

Quantity A	Quantity B
Amount Eliane earns in 9 hours	Amount Delphine earns in 12 hours