On your scratch paper write:

	x		y	x-y
B		B		
B		S		
S		B		
S		S		

Now just solve for $x - y$. When you're done, the biggest and smallest numbers are your answers.

	x		y	x-y
B	10	B	-1	11
B	10	S	-10	20
S	0	B	-1	1
S	0	S	-10	10

The range for $x - y$, therefore is $1 < x - y < 20$. Check your answer choices and eliminate.

WORKING WITH TWO VARIABLES

So far we've only dealt with simple equations that involving one variable. But on the GRE you'll sometimes have to deal with equations with two variables. Here's an example:

$$3x + 10y = 64$$

Here's How to Crack It

The important thing to note about this situation is that we cannot solve this equation. Why, you ask? The problem is that since there are two variables, there are many possible solutions to this equation and we have no way of knowing which solutions are correct. For example, the values $x = 8$ and $y = 4$ satisfy the equation. But so do the values $x = 10$ and $y = 3.4$. Which solutions are correct? We just don't know. In order to solve equations with two variables, we need two equations. Having two equations allows us to find definitive values for our variables.

You can't solve an equation with two variables unless you have a second equation.

$$3x + 10y = 64$$
$$6x - 10y = 8$$

When we're given two equations, we can combine them by adding or subtracting them. We do this so that we can cancel out one of the variables, leaving us with a simple equation with one variable. In this case, it's easier to add the two equations together:

$$\begin{array}{l} 3x + 10y = 64 \\ \underline{6x - 10y = 8} \\ 9x \qquad\;\; = 72 \end{array}$$

When we add these two equations we get $9x = 72$. This is a simple equation which we can solve to find $x = 8$. Once we've done that, we plug that value back into one of the equations and solve for the other variable. Substituting $x = 8$ into either equation gives us $y = 4$.

Try this one:

$$4x + 7y = 41$$
$$2x + 3y = 19$$

Here's How to Crack It

You might notice that if we add or subtract the two equations, we won't be left with one variable: Adding the two yields $6x + 10y = 60$. That doesn't help. Subtracting the equations leaves $2x + 4y = 22$. No help there, either. In cases like this one, you'll have to manipulate one of the equations so that subtracting or adding gets rid of one of the variables. In this case, let's multiply the second equation by 2:

$$2(2x + 3y) = 2(19)$$

This gives us the following:

$$4x + 6y = 38$$

Now we can subtract this equation from the first equation, yielding $y = 3$. If we substitute $y = 3$ into either of the equations we find that $x = 5$.

Quadratic Equations

Quadratic equations are special types of equations that involve, as the name suggests, four terms. Here is an example of a quadratic:

$$(x + 4)(x - 7)$$

In order to work with quadratics on the GRE, you must be familiar with two concepts: FOIL and factoring.

FOIL

When you see two sets of parentheses, all you have to do is remember to multiply every term in the first set of parentheses by every term in the second set of parentheses. Use FOIL to remember this method. FOIL stands for *first, outer, inner, last*—the four steps of multiplication. For example, if you see $(x + 4)(x + 3)$, you would multiply the first terms $(x \times x)$, the outer terms $(x \times 3)$, the inner terms $(4 \times x)$, and the last terms (4×3), as follows:

$$(x \times x) + (x \times 3) + (4 \times x) + (4 \times 3)$$
$$x^2 + 3x + 4x + 12$$
$$x^2 + 7x + 12$$

Quadratic Equations

There are three expressions of quadratic equations that can appear on the GRE. You should know them cold, in both their factored and unfactored forms. Here they are:

1. Factored form: $x^2 - y^2$ (the difference between two squares)
   Unfactored form: $(x + y)(x - y)$
2. Factored form: $(x + y)^2$
   Unfactored form: $x^2 + 2xy + y^2$
3. Factored form: $(x - y)^2$
   Unfactored form: $x^2 - 2xy + y^2$

This also works in the opposite direction. For example, if you were given $x^2 + 7x + 12 = 0$, you could solve it by breaking it down as follows:

$$(x + \ \ )(x + \ \ ) = 0$$

We know to use plus signs inside the parentheses because both the 7 and the 12 are positive. Now we have to think of two numbers that, when added together, give us 7, and when multiplied together, give us 12. Yep, they're 4 and 3:

$$(x + 4)(x + 3) = 0$$

Note that there are two solutions for x. So x can either be -4 or -3.

Let's see how this could be used on the GRE:

If x and y are positive integers, and if $x^2 + 2xy + y^2 = 25$, then $(x + y)^3 =$

- ○ 5
- ○ 15
- ○ 50
- ○ 75
- ○ 125

Here's How to Crack It

Problems like this one are the reason you have to memorize those quadratic equations. The equation in this question is Expression 2 from the previous page: $x^2 + 2xy + y^2 = (x + y)^2$. The question tells us that $x^2 + 2xy + y^2$ is equal to 25, which means that $(x + y)^2$ is also equal to 25. Think of $x + y$ as one unit that, when squared, is equal to 25. Since this question specifies that x and y are positive integers, what positive integer squared equals 25? Right, 5. So $x + y = 5$. The question is asking for $(x + y)^3$. In other words, what's 5 cubed, or $5 \times 5 \times 5$? It's 125. Choice (E).

Here's another one:

Quantity A	Quantity B
$(4 + \sqrt{6})(4 - \sqrt{6})$	10

- ○ Quantity A is greater.
- ○ Quantity B is greater.
- ○ The two quantities are equal.
- ○ The relationship cannot be determined from the information given.

Here's How to Crack It

First, eliminate choice (D)—we only have numbers here, so the answer can be determined. Now, Quantity A looks like a job for FOIL! Multiply the first terms, and you get 16. Multiply the outer terms and you get $-4\sqrt{6}$. Multiply the inner terms and you get $4\sqrt{6}$. Multiply the last terms and you get –6. So, we have $16-4\sqrt{6}+4\sqrt{6}-6$. Those two inner terms cancel each other out, and we're left with 16 – 6, or 10. What do you know? That's what we have in Quantity B, too! So, the answer is (C). You might also notice that Quantity A is common quadratic Expression 1 from the previous page: $(x+y)(x-y)=x^2-y^2$. Therefore, $(4+\sqrt{6})(4-\sqrt{6}) = 4^2-\sqrt{6}^2 = 16-6 = 10$.

Factoring

The process of factoring "undoes" the FOIL process. Here is a quadratic in its unfactored, or expanded, form:

$$x^2 - 10x + 24$$

From this point, we can factor a quadratic by taking the following steps:

1. Separate the x^2 into $(x\quad)(x\quad)$.
2. Find the factors of the third term that, when added or subtracted, yield the second term.
3. Figure out the signs (+/–) for the terms. The signs have to yield the middle number when added and the last term when multiplied.

If we apply these steps to the expression above, we first set up the problem by splitting x^2 into

$$(x\quad)(x\quad)$$

Next, write down the factors of the third term, 24. The factors are: 1 and 24, 2 and 12, 3 and 8, and 4 and 6. Of these pairs of factors, which contains two numbers that we can add or subtract to get the second term, 10? 4 and 6 are the only two that work. That gives us

$$(x\quad 4)(x\quad 6)$$

The final step is to figure out the signs. We need to end up with a negative 10 and a positive 24. If we add –6 and –4, we'll get –10. Similarly, if we multiply –6 and –4, we'll end up with 24. So the answer is

$$(x-4)(x-6)$$

Solving Quadratic Equations

ETS likes to use quadratic equations because they have an interesting quirk; often when you solve a quadratic equation, you get not one answer, but two. This property makes quadratic equations perfect ways for ETS to try to trick you.

Quadratic equations often have two solutions.

Here's an example:

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$$x^2 + 2x - 15 = 0$$

Quantity A	Quantity B
2	x

- ○ Quantity A is greater.
- ○ Quantity B is greater.
- ○ The two quantities are equal.
- ○ The relationship cannot be determined from the information given.

Here's How to Crack It

In order to solve a quadratic equation, the equation must be set equal to zero. Normally, this will already be the case on the GRE, as it is in this example. But if you encounter a quadratic equation that isn't set equal to zero, you must first manipulate the equation so that it is. Next you must factor the equation; otherwise you cannot solve it. So let's factor the quadratic equation in this example. We need to figure out the factors of 15 that we can add or subtract to give us 2. The only possible factors are 3 and 5. In order to get a negative 15 and a positive 2, we need to use 5 and –3. So that leaves us

$$(x-3)(x+5) = 0$$

Next, we're going to solve each of the two expressions within parentheses separately:

$$x - 3 = 0 \text{ and } x + 5 = 0$$

Thus, $x = 3$ and $x = -5$. This means that in this particular problem, the answer must be (D). If $x = 3$, then Quantity B is greater, but if $x = -5$ then Quantity A is greater.

Let's try another one:

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If $x^2 + 8x + 16 = 0$, then $x =$

Type your answer in the box.

Here's How to Crack It

Let's factor the equation. Start with (x) (x). Next, find the factors of 16 that add or subtract to 8. The factors of 16 are 1 and 16, 2 and 8, and 4 and 4. Of these pairs, only 4 and 4 work. Since we have a positive 8 and a positive 16, the signs for both numbers must be positive. Thus, we end up with: $(x + 4)(x + 4) = 0$. Now, we need to solve the equation. If $x + 4 = 0$, then $x = -4$. This is the number we'd enter into the text box on the GRE.

Simultaneous Equations

ETS will sometimes give you two equations and ask you to use them to find the value of a given expression. Don't worry, you won't need any math-class algebra; in most cases, all you will have to do to find ETS's answer is to add or subtract the two equations.

Here's an example:

If $5x + 4y = 6$ and $4x + 3y = 5$, then what does $x + y$ equal?

Here's How to Crack It

All you have to do is add together or subtract one from the other. Here's what we get when we add them:

$$\begin{array}{r} 5x+4y=6 \\ +\ 4x+3y=5 \\ \hline 9x+7y=11 \end{array}$$

A dead end. So let's try subtracting them:

$$\begin{array}{r} 5x+4y=6 \\ -\ 4x+3y=5 \\ \hline x+\ y=1 \end{array}$$

Bingo. The value of the expression $(x + y)$ is exactly what we're looking for. On the GRE, you may see the two equations written horizontally. Just rewrite the two equations, putting one on top of the other, then simply add or subtract them.

PLUGGING IN

Some of the hardest questions you might encounter on the GRE involve algebra. Algebra questions are generally difficult for two reasons. First, they are often complicated, multistep problems. Second, the answer choices often involve "distractor" choices. These are answer choices that look right, but they are actually wrong. They're designed to tempt you or to influence how you think about a problem.

If you don't like algebra, you're in luck. You don't have to do it. Plugging In will take even the hardest, messiest GRE problem and turn it into a simple arithmetic problem. It will never let you down, and it will never take more than a minute per problem.

Here are the steps:

Step 1: **Recognize the opportunity.** You can Plug In on any problem that has variables in the answer choices. The minute you see variables in the answers, even before you have read the problem, you know you can Plug In.

Why Plug In?

Plugging In is a powerful tool that can greatly enhance your math score, but you may be wondering why you should plug in when algebra works just fine. Here's why:

Plugging In converts algebra problems into arithmetic problems. No matter how good you are at algebra, you're better at arithmetic. Why? Because you use arithmetic every day, every time you go to a store, balance your checkbook, or tip a waiter. Chances are you rarely use algebra in your day to day activities.

Plugging In is more accurate than algebra. By Plugging In real numbers, you make the problems concrete rather than abstract. Once you're working with real numbers, it's easier to notice when and where you've messed up a calculation. It's much harder to see where you went wrong (or to even know you've done something wrong) when you're staring at a bunch of x's and y's.

The GRE allows the use of a calculator. A calculator can do arithmetic but it can't do algebra, so Plugging In allows you to take advantage of the calculator function.

ETS expects its students to attack the problems algebraically and many of the tricks and the traps built into the problem are designed to catch students who do the problems with algebra. By Plugging In, you'll avoid these pitfalls.

As you can see, there are a number of excellent reasons for Plugging In. Mastering this technique can have a significant impact on your score.

Step 2: **Engage the Hand.** You cannot solve Plugging In problems in your head. Even if it seems like an easy question of translating a word problem into an algebraic equation, remember that there are trap answer choices. When a question pops up, the minute you see variables, list your answer choices, A–E on your scratch paper.

Step 3: **Plug In.** If the question asks for "x apples," come up with a number for x. The goal here is to make your life easier, so Plug In something simple and happy, but avoid 1 or 0. If you Plug In a number and the math starts getting creepy (anything involving fractions or negative numbers is creepy), don't be afraid to just change the number you Plug In. Always label each variable on your scratch paper.

Step 4: **ID Target Number.** The Target Number is the value the problem asks you to solve for. Once you you've arrived at a Target Number, write it down on your scratch paper and circle it.

Step 5: **Check All Answer Choices.** Anywhere you see a variable, Plug In the number you have written down for that variable. Do any required math. The correct answer is the one that matches your target number. If more than one answer matches your target number, just Plug In a different number for your variables and test the remaining answer choices.

Plug In numbers that will make the math EASY.

Can I Just Plug In Anything?

You can Plug In any numbers you like, as long as they're consistent with any restrictions stated in the problem, but it's faster if you use easy numbers. What makes a number easy? That depends on the problem. In most cases, smaller numbers are easier to work with than larger numbers. Usually, it's best to start small, with 2, for example. Avoid 0 and 1; both 0 and 1 have special properties, which you'll hear more about later. You want to avoid these numbers because they will often make more than one answer choice appear correct. For example, if we Plug In 0 for a variable, then the answers $2x$, $3x$, and $5x$ would all equal 0. If you avoid these bad number choices, you should also avoid these bad situations. Also, do not Plug In any numbers that show up a lot in the question or answer choices.

Try this one. Read through the whole question before you start to Plug In numbers:

The price of a certain stock increased 8 points, then decreased 13 points, and then increased 9 points. If the stock price before the changes was x points, which of the following was the stock price, in points, after the changes?

- ○ $x - 5$
- ○ $x - 4$
- ○ $x + 4$
- ○ $x + 5$
- ○ $x + 8$

Here's How to Crack It

Let's use an easy number like 10 for the variable (write down "$x = 10$" on your scratch paper!). If the original price was 10, and then it increased 8 points, that's 18. Then it decreased 13 points, so now it's 5 (do everything out on the scratch paper—don't even add or subtract in your head). Then it increased 9 points, so now it's 14. So, it started at 10 and ended at 14. Circle 14 (our target answer) and Plug In 10 for every x in the answer choices. Which one gives you 14?

Don't skip steps! Use your scratch paper.

(A) $10 - 5 = 5$—Nope.
(B) $10 - 4 = 6$—Nope.
(C) $10 + 4 = 14$—Bingo!
(D) $10 + 5 = 15$—Nope.
(E) $10 + 8 = 18$—Nope.

Pretty easy, huh?

Good Numbers Make Life Easier

Small numbers aren't always the best choices for Plugging In, though. In a problem involving percentages, for example, 10 and 100 are good numbers to use. In a problem involving minutes or seconds, 30 or 120 are often good choices. (Avoid 60, however; it tends to cause problems.) You should look for clues in the problem itself to help you choose good numbers.

Always Plug In when you see variables in the answer choices!

Let's work through the following problem, using the steps above:

On the GRE, Plugging In is often safer, and easier, than doing the algebra.

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Mara has six more than twice as many apples as does Robert and half as many apples as does Sheila. If Robert has x apples, then, in terms of x, how many apples do Mara, Robert, and Sheila have?

- ○ $2x + 6$
- ○ $2x + 9$
- ○ $3x + 12$
- ○ $4x + 9$
- ○ $7x + 18$

What's your target number?

Here's How to Crack It

Step 1: **Identify the Opportunity.** You're sitting in your cubical at the Thompson Prometric Center and this question pops up. What do you see? The variable, x, is in both the question and the answer choices. Good, so what do you do?

Step 2: **Engage the Hand.** On the upper left-hand corner of your scratch paper, list answer choices (A) through (E). The answers aren't actually labeled (A) through (E), but this will help you to identify each one.

Step 3: **Plug In.** The problem tells us that Robert has x apples, so Plug In a number for x. Make it something nice and happy. Try 4. On your scratch paper, write $x = 4$.

Step 4: **ID Target Number.** The problem tells us that "Mara has six more than twice as many apples as does Robert." If Robert has 4 apples, then Mara must have 14. On your scratch paper, write $m = 14$. We are also told that Mara has "half as many apples as does Shelia." Ignoring the weird diction, that means that Shelia must have 28 apples. Write down $s = 28$. Now, what does the question ask you to find? It asks for the number of apples that Mara, Robert, and Shelia have. That's no problem; add the three up to come up with 46 apples. This is your target number. Write it down and circle it.

Step 5: **Check All Answer Choices.** You are allowed to perform only one mathematical function in your head at a time. Anything more than that leads to trouble. For the first answer choice, therefore, you can do $2x$ in your head; that's 8, but write down 8 + 6. You don't need to go any farther than that because this clearly will not add up to 46. Cross off choice (A). Choice (B) gives you 8 + 9. Cross that off. Choice (C) is 12 + 12. This is also too small, so cross it off. Choice (D) gives you 16 + 9. That gets you to 25, which is not your target number, so cross it off. Choice (E) is 28 + 18. Do this on your scratch paper or with the calculator. Do NOT do it in your head. It equals 46, which is your target number. Choice (E) is the correct answer.

(A) $2(4) + 6 = 14$	This is not 46, so eliminate it.
(B) $2(4) + 9 = 17$	No good either.
(C) $3(4) + 12 = 24$	Still not 46.
(D) $4(4) + 9 = 25$	This isn't 46 either.
(E) $7(4) + 18 = 46$	Bingo! This is your answer.

On the GRE, you can Plug In any time the question has variables in the answer choices. You can usually Plug In any number you wish, although you should always pick numbers that will be easy to work with. Some numbers can end up causing more trouble than they're worth.

When a problem has variables in the answer choices, PLUG IN!

When Plugging In, follow these rules:

1. Don't Plug In 0 or 1. These numbers, while easy to work with, have special properties.
2. Don't Plug In numbers that are already in the problem; this might confuse you as you work through it.
3. Don't Plug In the same number for multiple variables. For example, if a problem has x, y, and z in it, pick three different numbers to Plug In for the three variables.

Finally, Plugging In can be a powerful tool, but you must remember to always check all five answer choices when you plug in. In certain cases, two answer choices can yield the same target number. This doesn't necessarily mean you did anything wrong; you just hit some bad luck. Plug In some new numbers, get a new target and recheck the answers that worked the first time.

PLUGGING IN THE ANSWERS (PITA)

Some questions may not have variables in them but will try to tempt you into applying algebra to solve them. We call these Plugging In The Answers, or PITA for short. These are almost always difficult problems. Once you recognize the opportunity, however, they turn into simple arithmetic questions. In fact, the hardest part of these problems is often identifying them as opportunities for PITA. The beauty of these questions is that they take advantage of one of the inherent limitations of a multiple-choice test. ETS has actually given you the answers, and one of them must be correct. In fact, only one can work. The essence of this technique is to systematically Plug In The Answers into the question to see which answer choice works.

Here are the steps:

Step 1: **Recognize the Opportunity.** There are three ways to do this. The first triggers are the phrases "how much...," "how many...," or "what is the value of...." When you see one of these phrases in a question, you can Plug In The Answers. The second tip-off is specific numbers in the answer choices in ascending or descending order. The last tip-off is your own inclination. If you find yourself tempted to write your own algebraic formulas and to invent your own variables to solve the problem, it's a sure bet that you can just Plug In The Answer choices.

Step 2: **Engage the Hand.** The minute you recognize the opportunity, list the numbers in the answer choices in a column in the upper left-hand corner of your scratch paper.

Step 3: **Label the First Column.** What do these numbers represent? The question asks you to find a specific number. The answer choices are this number. At the top of the column, write down what these numbers represent.

Step 4: **Assume (C) to be Correct.** Choice (C) will always be the number in the middle. This the most efficient place to start because it will allow you to eliminate as many as three answer choices if it is wrong.

Step 5: **Create Your Spreadsheet.** Assuming the number in choice (C) is correct; use this number to work through the problem. It is always easier to understand the problem using a specific number. Work through the problem in bite-size pieces, and every time you have to do something with the number, make a new column. You can't have too many columns. Each column is a step in solving the problem.

Step 6: **Rinse and Repeat.** On single-answer multiple-choice questions, only one answer choice can work. If choice (C) is correct, you are done. If it is not correct, you may be able to identify if it is too big or too small. If it is too big, you can eliminate it and every answer choice that is bigger. This very quickly get's you down to a 50/50 shot. It also gives you a little spreadsheet specifically designed to calculate the correct answer. When you need to check the remaining answer choices, let the spreadsheet do the thinking for you. All you need to do is to fill in the cells. As soon as you find an answer choice that works, you're done.

Are you tempted to do algebra? Are there numbers in the answer choices? Plug In The Answers.

The following is an example of a PITA problem:

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An office supply store charged \$13.10 for the purchase of 85 paper clips. If some of the clips were 16 cents each and the remainder were 14 cents each, how many of the paper clips were 14-cent clips?

- ○ 16
- ○ 25
- ○ 30
- ○ 35
- ○ 65

Here's How to Crack It

Step 1: **Recognize the Opportunity.** The question asks "how many of the paper clips...." That's your first sign. Additionally, you have specific numbers in the answer choices in ascending order.

Step 2: **Engage the Hand.** The minute you recognize this as a PITA question, list your answer choices in a column on your scratch paper.

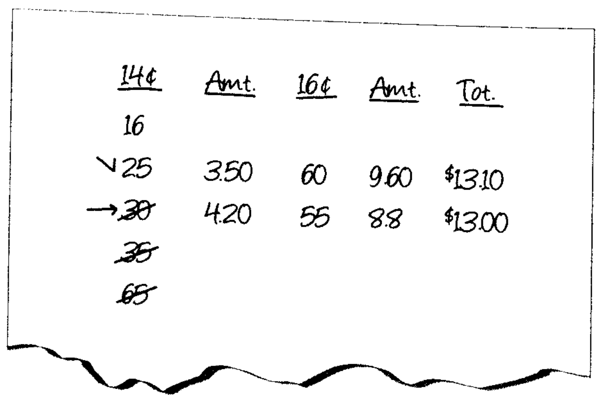
Step 3: **Label the First Column.** What do those answer choices represent? They are the number of 14-cent clips, so label this column 14¢.

Step 4: **Assume (C) to be Correct.** Start with choice (C) and assume that 30 of the clips were 14 cents each.

Step 5: **Create Your Spreadsheet.** If 30 of the clips were 14 cents each, then the purchaser would have spent \$4.20 on 14-cent clips. Label this column "amount spent." Now you know that there were 85 clips total, so if 30 of the clips were 14 cents each, there must have been 55 clips that were 16 cents each. Write down a 55 and label this column 16¢. The purchaser then spent \$8.80 on 16-cent clips. Write this down and label this column "amount spent." You can now calculate the total spent. 4.20 + 8.80 = 13.00. Write this down and label this column "total."

Step 6: **Rinse and Repeat.** You know that the purchaser spent \$13.10 on paper clips. If answer choice C were correct, then the purchaser would have spent only \$13.00 on paper clips. Since you know this is wrong, choice (C) cannot be correct. Cross it off. You also know that your total is too small. You need a greater portion of your clips to be the more expensive ones to get a higher total, so cross off choices (D) and (E). Now try choice (B). If 25 of the clips cost 14 cents each, the purchaser would have spent \$3.50. There must have been 60 clips that cost 16 cents each (85 – 25 = 60). Then, the purchaser would have spent \$9.60 on them. The total spent on clips, therefore, comes to \$13.10, and you're done.

Make sure to keep your hand moving, to write down all steps, and to use your calculator for simple arithmetic steps like multiplying and adding complex numbers. Here's what your scratch paper should look like after this problem:



On PITA questions, you can stop once you've found the correct answer; you don't have to check all five answer choices. Just make sure you write EVERYTHING down when doing these questions (and, indeed, all math questions).

PLUGGING IN ON QUANTITATIVE COMPARISON QUESTIONS

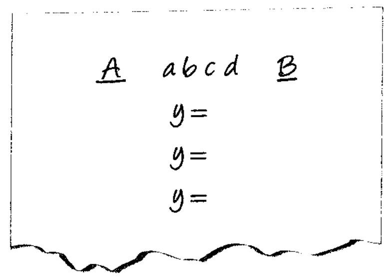
Quantitative Comparison questions with variables can be extremely tricky because the obvious answer is often wrong, whereas the correct answer may be a scenario most people would never think of. On the other hand, there is a simple set-up and approach that you can use that ensures that you get these questions right without taking too much time. As always, whenever you see variables, replace them with real numbers. On quant comp questions, however, it is crucial that you Plug In more than once and specifically that you Plug In all of the weird and obscure numbers that you would never use elsewhere. Always keep the nature of the answer choices in mind. Picking choice (A) means that you believe that the quantity in column A will *always* be bigger—*no matter what you Plug In.* Choice (B) means the column B will *always* be bigger—*no matter what you Plug In,* and so forth. To prove that one of these statements is true you have to Plug In every possible number that could change the outcome. Don't worry. We have a simple process to help figure out what to Plug In and how to track your progress as you do.

These questions often test your knowledge of the properties of fractions, zero, one, negatives, and other odd numbers.

Here are the steps:

Step 1: **Recognize the Opportunity.** The first six, seven, or eight questions of any math section will be quant comp. When a quant comp question pops up and you see variables, make your set up.

Step 2: **Engage the Hand.** The minute you see quant comp and variables make your set up in the upper left hand of your scratch paper. Your set-up looks like this:



Step 3: **Plug In and Eliminate.** Start with something nice and happy. Paying close attention to the rules the question gives you for what you are allowed to Plug In, and start with a simple, happy number. With a number for the variable, calculate the value in Quantity A and write it down. Then calculate the value in Quantity B and write it down. If Quantity A is bigger, eliminate choices (B) and (C). If Quantity B is bigger, eliminate (A) and (C). If they are both the same, eliminate choices (A) and (B). Note that you are already down to a 50/50 shot.

Step 4: **Rinse and Repeat**—There are still two answer choices left, so you're not done yet. The second time you Plug In, you want to try to get a different result. What can you Plug In the second time that messes with the problem? If you're not sure, use this simple check list: ZONE F. This stands for Zero, One, Negative, Extremely Big or Small, and Fractions. You won't always be allowed to Plug In all of these and rarely will you have to. Your goal is to eliminate choices (A), (B), and (C). If you Plug In everything on the checklist and (A), (B), or (C) is still standing, that's your answer.

The easiest way to solve most quant comp questions that involve variables is to Plug In, just as you would on word problems. But because answer choice (D) is always an option, you always have to make sure it isn't the answer. So...

Always Plug In at Least Twice in Quant Comp Questions

On quant comp, Plug In "normal" numbers, and eliminate two choices. Then Plug In "weird" numbers (zero, one, negatives, fractions, or big numbers) to try to disprove your first answer. If different numbers give you different answers, you've proved that the answer is (D).

Plugging In on quant comp questions is just like Plugging In on "must be" problems. The reason for this is (D). On quant comp questions, it's not enough to determine whether one quantity is sometimes greater than, less than, or equal to the other; you have to determine whether it *always* is. If different numbers lead to different answers, then the correct answer is (D). To figure out if one quantity is always bigger, you have to Plug In weird numbers to account for all possible situations.

What makes certain numbers weird? They behave in unexpected ways when added, multiplied, or raised to powers. Here are some examples:

- 0 times any number is 0.
- 0^2 is 0.
- 1^2 is 1.
- $\left(\frac{1}{2}\right)^2$ is less than $\frac{1}{2}$.
- $(-2)(-2)$ is 4.
- A negative number squared is positive.
- Really big numbers (100, 1,000) can make a really big difference in your answer.

Here's how it works:

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Quantity A	Quantity B
$2x^3$	$4x^2$

- Quantity A is greater.
- Quantity B is greater.
- The two quantities are equal.
- The relationship cannot be determined from the information given.

Here's How to Crack It

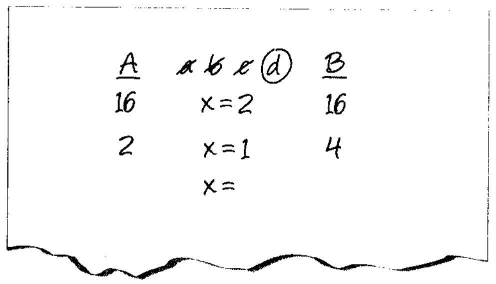
Step 1: **Recognize the Opportunity.** First you see quant comp. Second, you see variables. It takes all of three seconds to recognize a quant comp Plug In. You don't even have to understand the problem at this point. Just recognize the opportunity.

Step 2: **Engage the Hand.** The minute you recognize this as a quant comp Plug In, make your set-up on your scratch paper. List "$x =$" three times down the middle.

Step 3: **Plug In.** Let's start with something nice and happy, like 2. Write down 2 next to your first x. When $x = 2$, the quantity in column A is 16 ($2 \cdot 2^3$), and the quantity in column B is also 16 ($4 \cdot 2^2$). Since you have followed the rules and both columns are the same, neither (A) nor (B) can be the answer. Cross them off. Note that you haven't worked very hard yet, haven't spent much time at all, and you are already down to a 50/50 shot.

Step 4: **Rinse and Repeat.** Now try something different for x. What if $x = 1$? The quantity in column A will be 2, and the quantity in column B will be 4. In this case, they are not the same, so choice (C) cannot be the correct answer. Cross it off. Only choice (D) is left, so you're done.

Here is what your scratch paper should look like:



You might also have noticed that Plugging In $x = 0$ would also yield different results. On quant comp questions, ETS hopes you'll forget to consider what happens when you use numbers such as 0, 1, fractions, and negatives. Therefore, when Plugging In, make sure to use the following numbers whenever possible:

Zero
One
Negatives
Extreme Values
Fractions

We call these weird numbers the ZONE-F numbers. Make sure you use them aggressively on quant comp problems because they can radically affect the relationship between the two quantities.

Phew. Now we've covered the basics of mathematical operations; hopefully a lot of this material came back to you as we went through it, but if not don't worry! You'll have plenty of opportunities to refresh your memory of this material as you read through the next two chapters and work the problems you see in the drills.

In the next chapter we'll look at some everyday math topics that are tested on the GRE, so practice the techniques in the drill that follows, and move on!

Numbers and Equations Drill

Ready to try out your new skills? Give this drill a shot and then check your answers in Part V.

1 of 10

Which of the following is equal to 10 ?

*Indicate **all** possible values.*

☐ $\frac{2}{3} \times 33 - 12$

☐ $\frac{2}{3} \times 51 - 24$

☐ $33 - 22 \times 1\frac{1}{2}$

☐ $51 \div (17 \times 3) + 9$

☐ $(51 \div 17) \times 3 + 9$

Click on your choice(s).

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$$a + 7 = 23$$
$$b - a = -10$$

Quantity A	**Quantity B**
b	4

○ Quantity A is greater.

○ Quantity B is greater.

○ The two quantities are equal.

○ The relationship cannot be determined from the information given.

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If $3^3 \times 9^{12} = 3^x$, what is the value of x ?

☐

Click on the answer box, then type in a number. Backspace to erase.

4 of 10

If $x - y > 6$ and $2x + y > 30$, then which one represents all possible values of x ?

○ $x > 6$

○ $6 < x < 18$

○ $x > 12$

○ $12 < x < 36$

○ $x > 36$

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A merchant sells three different sizes of canned tomatoes. A large can costs as much as 5 medium cans or 7 small cans. If a customer buys an equal number of small and large cans of tomatoes for the exact amount of money that would buy 200 medium cans, how many small cans will she buy?

○ 35

○ 45

○ 72

○ 199

○ 208

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If $(x + y)^2 = 16$ and $(x - y)^2 = 9$, what is one possible value of $(x^2 - y^2)$?

☐

Click on the answer box, then type in a number. Backspace to erase.

When the integer a is multiplied by 3, the result is 4 less than 6 times the integer b. Therefore, $a - 2b$ is

- ○ -12
- ○ $-\frac{4}{3}$
- ○ $-\frac{3}{4}$
- ○ $\frac{4}{3}$
- ○ 12

Quantity A	Quantity B
$\dfrac{\sqrt{12}}{\sqrt{5}-\sqrt{2}}$	$\dfrac{\sqrt{2}+\sqrt{5}}{\sqrt{27}}$

- ○ Quantity A is greater.
- ○ Quantity B is greater.
- ○ The two quantities are equal.
- ○ The relationship cannot be determined from the information given.

$$\frac{a+\frac{b}{c}}{\frac{d}{e}}$$

If the value of the expression above is to be halved by doubling exactly one of the five numbers a, b, c, d, or e, which should be doubled?

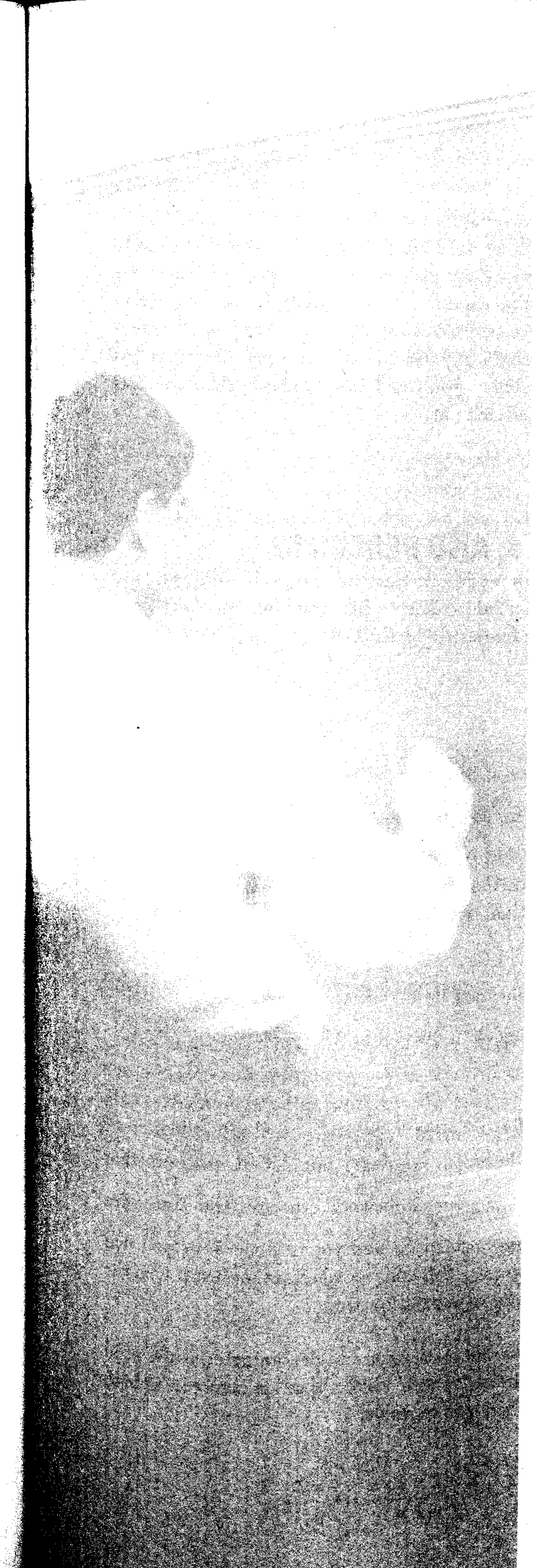
- ○ a
- ○ b
- ○ c
- ○ d
- ○ e

If $x = 3a$ and $y = 9b$, then all of the following are equal to $2(x + y)$ EXCEPT

- ☐ $2(3a) + 9b$
- ☐ $12a + 9b$
- ☐ $6a + 18b$
- ☐ $6(a + 3b)$
- ☐ $6a + 21b$

Summary

- Digits are the numbers that make up other numbers. Numbers include whole numbers, fractions, negative numbers, and weird values like the square root of 2. Integers are numbers with no decimal or fractional part.
- Positive numbers are greater than zero and negative numbers less than zero. The number zero is neither positive nor negative.
- Even numbers are divisible by 2; odd numbers aren't. Only integers can be even or odd.
- A factor divides evenly into a number. A multiple is a number that a certain number is a factor of. Every number is a factor and a multiple of itself.
- The order of operations is PEMDAS.
- An exponent is shorthand for repeated multiplication. When in doubt on exponent problems, expand them out.
- The Golden Rule of Equations: Whatever you do to one side of the equation, you must do to the other.
- With inequalities you have to flip the sign when multiplying or dividing by a negative number.
- In order to solve an equation with two variables you need two equations. Stack them up and add or subtract to cancel out one of the variables.
- Use the FOIL process to expand quadratics. To solve a quadratic equation, set it equal to zero and factor.
- Plugging In converts algebra problems to arithmetic problems. Plug In by replacing variables in the question with real numbers or by working backwards from the answer choices provided.
- Use the ZONE-F numbers on tricky quant comp questions with variables.



Chapter 10
Real World Math

Real world math is our title for the grab bag of math topics that will be heavily tested on the GRE. This chapter details a number of important math concepts, many of which you've probably used at one point or another in your daily adventures, even if you didn't recognize that you were. After completing this chapter you'll have brushed up on important topics such as fractions, percents, ratios, proportions, and average. You'll also learn some important Princeton Review methods for organizing your work and efficiently and accurately answering questions on these topics.

The math on the GRE is supposed to reflect the math you use in your day-to-day activities.

EVERYDAY MATH

As we've mentioned, when ETS reconfigured the GRE, one of its goals was to make the Math section reflect more of the kind of math that a typical graduate school student would use. Another of their goals was to test more of what it calls "real-life" scenarios. You can therefore expect the math questions on the GRE to heavily test things such as fractions, percents, proportions, averages, and ratios—mathematical concepts that are theoretically part of your everyday life. Regardless of whether that's true of your daily life or not, you'll have to master these concepts in order to do well on the GRE Math section.

FRACTIONS, DECIMALS, AND PERCENTS

In the previous chapter we spent most of our time working with integers. Now we'll expand our discussion to include concepts like fractions, decimals, and percents—all of which will appear frequently on the GRE.

Fractions

A fraction expresses a specific piece of information; namely the number of parts out of a whole. In the fraction $\frac{2}{3}$, for instance, the top part, or numerator, tells us that we have 2 parts, while the bottom part of the fraction, the denominator, indicates that the whole, or total, consists of 3 parts. We use fractions whenever we're dealing with a quantity that's less than one.

Fractions are important on the GRE. Make sure you're comfortable with them.

Notice that the fraction bar is simply another way of expressing division. Thus, the fraction $\frac{2}{3}$ is just expressing the idea of "2 divided by 3."

Reducing and Expanding Fractions

Fractions express a relationship between numbers, not actual amounts. For example, saying that you did $\frac{1}{2}$ of your homework expresses the same idea whether you had 10 pages of homework to do and you've done 5, or you had 50 pages to do and you've done 25 pages. This concept is important because on the GRE you'll frequently have to reduce or expand fractions.

To reduce a fraction, simply express the numerator and denominator as the products of their factors. Then cross out, or "cancel," factors that are common to both the numerator and denominator. Here's an example:

$$\frac{16}{20}=\frac{2\times2\times2\times2}{2\times2\times5}=\frac{\cancel{2}\times\cancel{2}\times2\times2}{\cancel{2}\times\cancel{2}\times5}=\frac{2\times2}{5}=\frac{4}{5}$$

You can achieve the same result by dividing the numerator and denominator by the factors that are common to both. In the example you just saw, you might realize that 4 is a factor of both the numerator and the denominator. That is, both the numerator and the denominator can be divided evenly (without remainder) by 4. Doing this yields the much more manageable fraction $\frac{4}{5}$.

When you confront GRE math problems that involve big fractions, always reduce them before doing anything else.

> Remember: You can only reduce across a multiplication sign.

Look at each of the following fractions:

$$\frac{1}{4}\quad\frac{2}{8}\quad\frac{6}{24}\quad\frac{18}{72}\quad\frac{90}{360}\quad\frac{236}{944}$$

What do you notice about each of these fractions? They all express the same information! Each of these fractions expresses the relationship of "1 part out of 4 total parts."

Adding and Subtracting Fractions

Adding and subtracting fractions that have a common denominator is easy—you just add the numerators and put the sum over the common denominator. Here's an example:

$$\frac{1}{10}+\frac{2}{10}+\frac{4}{10}=$$

$$\frac{1+2+4}{10}=\frac{7}{10}$$

> **Why Bother?**
> You may be wondering why, if the GRE allows the use of a calculator, you should bother learning how to add or subtract fractions or to reduce them or even know any of the topics covered in the next few pages. While it's true that you can use a calculator for these tasks, for many problems it's actually slower to do the math with the calculator than without. Scoring well on the GRE Math section requires a fairly strong grasp of the basic relationships among numbers, fractions, percents, and so on, so it's in your best interest to really understand these concepts rather than to rely on your calculator to get you through the day. In fact, if you put in the work now, you'll be surprised at how easy some of the problems become, especially when you don't have to refer constantly to the calculator to perform basic operations.

In order to add or subtract fractions that have different denominators, you need to start by finding a common denominator. You may remember your teachers from grade school imploring you to find the "lowest common denominator." Actually, any common denominator will do, so find whichever one you find most comfortable working with.

$$\frac{7}{8}-\frac{5}{12}=\frac{21}{24}-\frac{10}{24}=\frac{11}{24}$$

Here, we expanded the fraction $\frac{7}{8}$ into the equivalent fraction $\frac{21}{24}$ by multiplying both the numerator and denominator by 3. Similarly, we converted $\frac{5}{12}$ to $\frac{10}{24}$ by multiplying both denominator and numerator by 2. This left us with two fractions that had the same denominator, which meant that we could simply subtract their numerators.

When adding and subtracting fractions, you can also use a technique we call the Bowtie. The Bowtie method accomplishes exactly what we just did in one fell swoop. To use the Bowtie, first multiply the denominators of each fraction. This gives us a common denominator. Then multiply the denominator of each fraction by the numerator of the other fraction. Take these numbers and add or subtract them—depending on what the question asks you to do—to get the numerator of the answer. Then reduce if necessary.

The Bowtie method is a convenient shortcut to use when adding and subtracting fractions.

$$\frac{2}{3}+\frac{3}{4}=$$

$$\overset{8}{\frac{2}{3}}\;\overset{9}{\frac{3}{4}}=\frac{8}{12}+\frac{9}{12}=\frac{17}{12}$$

and

$$\frac{2}{3}-\frac{3}{4}=$$

$$\overset{8}{\frac{2}{3}}\;\overset{9}{\frac{3}{4}}=\frac{8}{12}-\frac{9}{12}=-\frac{1}{12}$$

Multiplying Fractions

There's nothing tricky about multiplying fractions: All you do is multiply straight across—multiply the first numerator by the second numerator and the first denominator by the second denominator. Here's an example:

$$\frac{4}{5}\times\frac{10}{12}=\frac{40}{60}$$

At this point, we'd probably want to reduce our fraction. When multiplying fractions, we can make our lives easier by reducing before we multiply. We do this once again by dividing out common factors.

Multiplying fractions is a snap: just multiply straight across, numerator times numerator and denominator times denominator.

$$\frac{4}{5}\times\frac{10}{12}=\frac{4}{5}\times\frac{5}{6}$$

Also remember that when we're multiplying fractions, we can even reduce diagonally; as long as we're working with a numerator and a denominator of opposite fractions; they don't have to be in the same fraction. So we end up with

$$\frac{4}{5}\times\frac{5}{6}=\frac{2}{1}\times\frac{1}{3}=\frac{2}{3}$$

Of course, you get the same answer either way, so attack fractions in whatever fashion you find easiest.

Dividing Fractions

Dividing fractions is just like multiplying fractions, with one crucial difference: Before you multiply, you have to turn the second fraction upside down (that is, put its denominator over its numerator, or to use fancy math lingo, find its reciprocal). In some cases, you can also reduce before you multiply. Here's an example:

$$\frac{2}{3}\div\frac{4}{5}=\frac{2}{3}\times\frac{5}{4}=\frac{1}{3}\times\frac{5}{2}=\frac{5}{6}$$

ETS sometimes gives you problems that involve fractions whose numerators or denominators are themselves fractions. These problems might look intimidating, but if you're careful, you won't have any trouble with them. All you have to do is remember what we said about a fraction being shorthand for division. Always rewrite the expression horizontally. Here's an example:

$$\frac{7}{\frac{1}{4}} = 7 \div \frac{1}{4} = \frac{7}{1} \times \frac{4}{1} = \frac{28}{1} = 28$$

Comparing Fractions

The GRE might also present you with math problems that require that you to compare two fractions and decide which is larger, especially on quant comp questions. There are a couple of ways to accomplish this. One is to find equivalent fractions that have a common denominator. This works with simpler fractions, but on some problems the common denominator might be hard to find or hard to work with.

As an alternative, you can use a variant of the Bowtie technique. In this variant, you don't have to multiply the denominators, just the denominators and the numerators. The fraction with the larger product in its numerator is the bigger fraction. Let's say we had to compare the following fractions:

$$\frac{3}{7} \qquad \frac{7}{12}$$

$$\begin{array}{cc} 36 & 49 \\ \frac{3}{7} & \frac{7}{12} \end{array}$$

You can also use the calculator feature to change the fractions into decimals.

Multiplying the first denominator by the second numerator gives us 49. This means the numerator of the second fraction $\left(\frac{7}{12}\right)$ will be 49. Multiplying the second denominator by the first numerator gives us 36, which means the first fraction will have a numerator of 36. If 49 is bigger than 36, $\frac{7}{12}$ is bigger than $\frac{3}{7}$. Remember that when you use this method, it's the numerators that matter.

Comparing More Than Two Fractions

You may also be asked to compare more than two fractions. On these types of problems, don't waste time trying to find a common denominator for all of them. Simply use the Bowtie to compare two of the fractions at a time.

Here's an example:

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Which of the following statements is true?

- ○ $\frac{3}{8} < \frac{2}{9} < \frac{4}{11}$
- ○ $\frac{2}{5} < \frac{3}{7} < \frac{4}{13}$
- ○ $\frac{4}{13} < \frac{2}{5} < \frac{3}{7}$
- ○ $\frac{3}{7} < \frac{3}{8} < \frac{2}{5}$
- ○ $\frac{2}{9} < \frac{3}{7} < \frac{3}{8}$

Here's How to Crack It

As you can see, it would be a nightmare to try to find common denominators for all these funky fractions, so instead we'll use the Bowtie method. Simply multiply the denominators and numerators of a pair of fractions and note the results. For example, to check answer choice (A), we first multiply 8 and 2, which gives us a numerator of 16 for the fraction $\frac{2}{9}$. But multiplying 9 and 3 gives us a numerator of 27 for the first fraction. This means that $\frac{3}{8}$ is bigger than $\frac{2}{9}$, and we can eliminate choice (A), because the first part of it is wrong. Here's how the rest of the choices shape up:

- ○ $\frac{2}{5} < \frac{3}{7} < \frac{4}{13}$ Compare $\frac{3}{7}$ and $\frac{4}{13}$; $\frac{3}{7}$ is larger.
- ○ $\frac{4}{13} < \frac{2}{5} < \frac{3}{7}$ These fractions are in order.
- ○ $\frac{3}{7} < \frac{3}{8} < \frac{2}{5}$ $\frac{3}{7}$ is larger than $\frac{3}{8}$.
- ○ $\frac{2}{9} < \frac{3}{7} < \frac{3}{8}$ $\frac{3}{7}$ is larger than $\frac{3}{8}$.

Make sure you are doing all of this work in an organized fashion on your scratch paper.

Converting Mixed Numbers into Fractions

A **mixed number** is a number that is represented as an integer and a fraction, such as $2\frac{2}{3}$. In most cases on the GRE, you should get rid of mixed fractions by converting them to fractions. How do you do this? By multiplying the denominator of the fraction by the integer, then adding that number to the numerator, and then putting the whole thing over the denominator. In other words, for the fraction above we would get $\frac{3\times2+2}{3}$ or $\frac{8}{3}$.

The result, $\frac{8}{3}$, is equivalent to $2\frac{2}{3}$. The only difference is that $\frac{8}{3}$ is easier to work with in math problems. Also, answer choices are usually not in the form of mixed numbers.

Decimals

Decimals are just fractions in disguise. Basically, decimals and fractions are two different ways of expressing the same thing. Every decimal can be written as a fraction, and every fraction can be written as a decimal. For example, the decimal .35 can be written as the fraction $\frac{35}{100}$: These two expressions, .35 and $\frac{35}{100}$, have the same value.

To turn a fraction into its decimal equivalent, all you have to do is divide the numerator by the denominator. Here, for example, is how you would find the decimal equivalent of $\frac{3}{4}$:

$$\frac{3}{4} = 3 \div 4 = 4\overline{)3.00}\quad 0.75$$

Try this problem:

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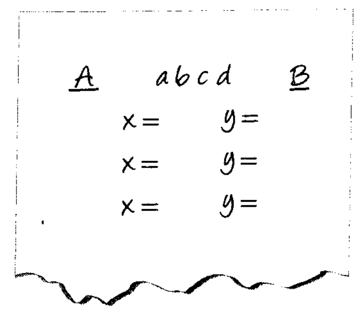
$$7 < x < 8$$
$$y = 9$$

Quantity A	Quantity B
$\frac{x}{y}$	.85

- Quantity A is greater.
- Quantity B is greater.
- The two quantities are equal.
- The relationship cannot be determined from the information given.

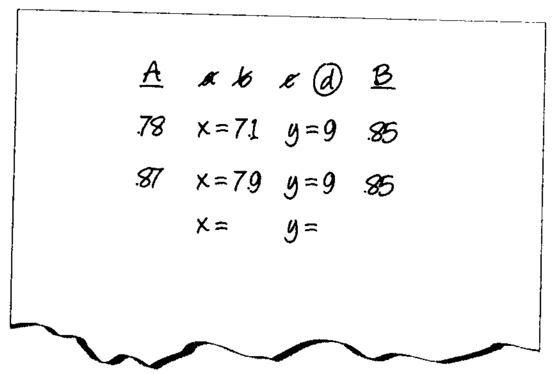
Here's How to Crack It

So, you're sitting at your cubical at the Thompson Prometric Center and this problem pops up. What do you see? Before we even talk about fractions, the first thing you should note is that this is a quant comp with variables. With your hand, on your scratch paper, make your set-up. It should look like this:



Now they've told us that x is going to be seven point something. Try Plugging In the smallest value you can think of for x. Write down $x = 7.1$ and $y = 9$. The value in Quantity A is .78. The value in Quantity B is .85. Quantity B is bigger, so eliminate choices (A) and (C). Now try making x as big as you can make it. Write down $x = 7.9$ and $y = 9$. The value in column A is .87 and the value in Quantity B is .85. Quantity A is bigger so eliminate choice B, and you're done. The answer is (D).

Your scratch paper should look like this:



Comparing Decimals

Which is larger: 0.00099 or 0.001? ETS loves this sort of problem. You'll never go wrong, though, if you follow these easy steps.

- Line up the numbers by their decimal points.
- Fill in the missing zeros.

Here's how to answer the question we just asked. First, line up the two numbers by their decimal points.

0.00099
0.001

Now fill in the missing zeros.

0.00099
0.00100

Can you tell which number is larger? Of course you can. 0.00100 is larger than 0.00099, because 100 is larger than 99.

Digits and Decimals

Remember our discussion about digits, earlier? Well, sometimes the GRE will ask you questions about digits that fall after the decimal point as well. Suppose you have the number 0.584.

- 0 is the units digit.
- 5 is the tenths digit.
- 8 is the hundredths digit.
- 4 is the thousandths digit.

Percentages

The final member of our numbers family is percents. A percent is just a special type of fraction, one that always has 100 as the denominator. Percent literally means "per 100" or "out of 100" or "divided by 100." If your best friend finds a dollar and gives you 50¢, your friend has given you 50¢ out of 100, or $\frac{50}{100}$ of a dollar, or 50 percent of the dollar. To convert fractions to percents, just expand the fraction so it has a denominator of 100. For example:

$$\frac{3}{5} = \frac{60}{100} = 60\%$$

For the GRE, you should memorize the following percentage-decimal-fraction equivalents. Use these friendly fractions and percentages to eliminate answer choices that are way out of the ballpark.

$0.01 = \frac{1}{100} = 1\%$ $\qquad$ $0.1 = \frac{1}{10} = 10\%$

$0.2 = \frac{1}{5} = 20\%$ $\qquad$ $0.25 = \frac{1}{4} = 25\%$

$0.333... = \frac{1}{3} = 33\frac{1}{3}\%$ $\qquad$ $0.4 = \frac{2}{5} = 40\%$

$0.5 = \frac{1}{2} = 50\%$ $\qquad$ $0.6 = \frac{3}{5} = 60\%$

Percents are another very common topic on the GRE.

$0.666... = \frac{2}{3} = 66\frac{2}{3}\%$ $\quad$ $0.75 = \frac{3}{4} = 75\%$

$0.8 = \frac{4}{5} = 80\%$ $\quad$ $1.0 = \frac{1}{1} = 100\%$

$2.0 = \frac{2}{1} = 200\%$

Converting Decimals to Percentages

In order to convert decimals to percents, just move the decimal point two places to the right. This turns 0.8 into 80 percent, 0.25 into 25 percent, 0.5 into 50 percent, and 1 into 100 percent.

Translation

These translations apply to any word problem, not just percent problems.

One of the best tricks for handling percentages in word problems is knowing how to translate them into an equation that you can manipulate. Use the following table to help you translate percentage word problems into equations you can work with.

Word	Equivalent Symbol
percent	/100
is	=
of, times, product	×
what (or any unknown value)	any variable (*x*, *k*, *b*)

Here's an example:

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56 is what percent of 80 ?

- 66%
- 70%
- 75%
- 80%
- 142%

Here's How to Crack It

To solve this problem, let's translate the question and then solve for the variable. So, "56 is what percent of 80," in math speak, is equal to

$$56 = \frac{x}{100}(80)$$

$$56 = \frac{80x}{100}$$

Don't forget to reduce: $56 = \frac{5}{4}x$

Now multiply both sides of the equation by $\frac{5}{4}$.

$$\left(\frac{5}{4}\right)\left(\frac{56}{1}\right) = \left(\frac{5}{4}\right)\left(\frac{4x}{5}\right)$$

$$(5)(14) = x$$

$$70 = x$$

That's answer choice (B). Did you notice choice (E)? Because 56 is less than 80, the answer would have to be less than 100 percent, so 142 percent is way too big, and you could have eliminated it from the get-go by Ballparking.

Let's try a quant comp example.

5 of 20

5 is r percent of 25
s is 25 percent of 60

Quantity A	Quantity B
r	s

- Quantity A is greater.
- Quantity B is greater.
- The two quantities are equal.
- The relationship cannot be determined from the information given.

Here's How to Crack It

First translate the first statement.

$$5 = \frac{r}{100}(25)$$

$$5 = \frac{25r}{100}$$

$$5 = \frac{r}{4}$$

$$(4)(5) = \left(\frac{r}{4}\right)(4)$$

$$20 = r$$

That takes care of Quantity A. Now translate the second statement.

$$s = \frac{25}{100}(60)$$

$$s = \frac{1}{4}(60)$$

$$s = 15$$

That takes care of Quantity B. The answer is (A).

Percentage Increase/Decrease

On percent increase problems, the original is always the smaller number. On percent decrease problems, the original is the larger number.

To find the percentage by which something has increased or decreased, use the following formula.

$$\text{Percent Change} = \frac{\text{Difference}}{\text{Original}} \times 100$$

The "difference" is simply what you get when you subtract the smaller number from the larger number. The "original" is whichever number you started with. If the question asks you to find a percent increase, then the original number will be the smaller number. If the question asks you to find a percent decrease, then the original number will be the larger number.

Here's an example.

What number goes on the bottom of the fraction?

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Vandelay Industries reported a \$6,000 profit over the three-month period from March to May of the current year. If, over the previous three-month period, Vandelay Industries realized a \$3,500 profit, by approximately what percent did its profit increase?

- ○ 25%
- ○ 32%
- ○ 42%
- ○ 55%
- ○ 70%

Here's How to Crack It

Let's use the percent change formula we just learned. The first step is to find the difference between the two numbers. The initial profit was \$3,500 and the final profit is \$6,000. The difference between these two numbers is: 6,000 – 3,500 = 2,500. Next, we need to divide this number by the original, or starting, value.

One way to help you figure out what value to use as the original value is to check to see whether you're dealing with a percent increase or a percent decrease question. Remember that on a percent increase question, you should always use the smaller of the two numbers as the denominator and that on percent decrease you need to use the larger of the two numbers as the denominator. Because here we want to find the percent increase, the number we want to use for our denominator is 3,500. So our percent increase fraction looks like this: $\frac{2{,}500}{3{,}500}$. We can reduce this down to $\frac{25}{35}$ by dividing by 100, and reduce even further by dividing by 5. This leaves us with $\frac{5}{7}$, which is approximately 70% (remember, the fraction bar means divide, so if you divide 5 by 7, you'll get .71). Thus, choice (E) is the answer.

Here's another question.

13 of 20

Model	Original Price	Sale Price
A	$12,000	$9,500
B	$16,000	$13,000
C	$10,000	$7,500
D	$17,500	$13,000
E	$20,000	$15,500
F	$22,000	$16,000

The table above shows the original price and the sale price of six different models of cars. Which models of car would a consumer have to buy to get at least a 25% discount? Select all that apply.

☐ A

☐ B

☐ C

☐ D

☐ E

☐ F

Here's How to Crack It

First list A, B, C, D, E, and F in a column in the upper left corner of your scratch paper. You are asked to identify a 25% change or greater between the two numbers. You know the formula for this. It is diff/original × 100. Using your calculator, subtract 9,500 from 12,000. You should get 2,500. This is the difference. Divide it by the original, 12,000, to get 0.2. You don't even need to multiply by 100. You know that this is 20%, which is less than 25%, so cross it off on your scratch paper. Try the next one. 16,000 – 13,000 = 3,000. Divide by 16,000. Too small. Cross it off. Repeat this process for each of the answer choices. Choices (C), (D), and (F) all work.

PLUGGING IN ON FRACTION AND PERCENT PROBLEMS

Now that you've become familiar with fractions and percents, we'll show you a neat trick. When you come to regular multiple-choice questions, or multiple choice, multiple answers, that involve fractions or percents, you can simply Plug In a number and work through the problem using that number. This approach works even when the problem doesn't have variables in it. Why? Because, as you know, fractions and percents only express a relationship between numbers—the actual numbers don't matter. For example, look at the following problem:

Plugging In on fraction and percent problems is a great way to make your life easier.

17 of 20

A recent survey of registered voters in City x found that $\frac{1}{3}$ of the respondents support the mayor's property tax plan. Of those who did not support the mayor's plan, $\frac{1}{8}$ indicated they would not vote to reelect the mayor if the plan was implemented. Of all the respondents, what fraction indicated that it would not vote for the mayor if the plan is enacted?

○ $\frac{1}{16}$

○ $\frac{1}{12}$

○ $\frac{1}{6}$

○ $\frac{1}{3}$

○ $\frac{2}{3}$

What important information is missing from the problem?

Here's How to Crack It

Even though there are no variables in this problem, we can still Plug In. On fraction and percent problems, ETS will often leave out one key piece of information: the total. Plugging In for that missing value will make your life much easier. What crucial information did ETS leave out of this problem? The total number of respondents. So let's Plug In a value for it. Let's say that there were 24 respondents to the survey. 24 is a good number to use because we'll have to work with $\frac{1}{3}$ and

$\frac{1}{8}$, so we want a number that's divisible by both those fractions. Working through the problem with our number, we see that $\frac{1}{3}$ of the respondents support the plan. $\frac{1}{3}$ of 24 is 8, so that means 16 people do not support the plan. Next, the problem says that $\frac{1}{8}$ of those who do not support the plan will not vote for the mayor. $\frac{1}{8}$ of 16 is 2, so 2 people won't vote for the mayor. Now we just have to answer the question: of all respondents, how many will not vote for the mayor? Well, there were 24 total respondents and we figured out that 2 aren't voting. So that's $\frac{2}{24}$ or $\frac{1}{12}$. Answer choice (B) is the one we want.

RATIOS AND PROPORTIONS

If you're comfortable working with fractions and percents, you'll be comfortable working with ratios and proportions, because ratios and proportions are simply special types of fractions. Don't let them make you nervous. Let's look at ratios first and then deal with proportions.

What Is a Ratio?

A ratio is just another type of fraction.

Recall that a fraction expresses the relationship of a part to the whole. A ratio expresses a different relationship: part to part. Imagine yourself at a party with 8 women and 10 men in attendance. What fraction of the partygoers are female? $\frac{8}{18}$, or 8 women out of a total of 18 people at the party. But what's the ratio of women to men? $\frac{8}{10}$, or, as ratios are more commonly expressed, 8 : 10. You can reduce this ratio to 4 : 5, just like you would a fraction.

On the GRE, you may see ratios expressed in several different ways:

Ratios can be expressed in these ways.

- $x : y$
- the ratio of x to y
- x is to y

In each case, the ratio is telling us the relationship between parts of a whole.

Every Fraction Can Be a Ratio, and Vice Versa

Every ratio can be expressed as a fraction. A ratio of 1 : 2 means that there's either a total of three things or a multiple of three, and the fraction $\frac{1}{2}$ means "1 out of 2."

On the GRE, you may see ratios expressed in several different ways:

$x : y$
the ratio of x to y
x is to y

Treat a Ratio Like a Fraction

Anything you can do to a fraction you can also do to a ratio. You can cross-multiply, find common denominators, reduce, and so on.

Find the Total

The key to dealing with ratio questions is to find the whole, or the total. Remember, a ratio only tells us about the parts, not the total. In order to find the total, add up the numbers in the ratio. A ratio of 2 : 1 means that there are three total parts. A ratio of 2 : 5 means that we're talking about a total of 7 parts. And a ratio of 2 : 5 : 7 means there are 14 total parts. Once you have a total you can start to do some fun things with ratios.

Like a fraction, a ratio expresses a relationship between numbers.

For example, let's say you have a handful of pennies and nickels. If you have 30 total coins and the pennies and nickels are in a 2 :1 ratio, how many pennies do you have? The total for our ratio is 3, meaning that out of every 3 coins, there are 2 pennies and 1 nickel. So if there are 30 total coins, there must be 20 pennies and 10 nickels. Notice that $\frac{20}{10}$ is the same as $\frac{2}{1}$, is the same as 2 : 1!

The minute you see the word "ratio," draw your box on your scratch paper.

When working with ratios, there's an easy way not only to keep track of the numbers in the problem but also to quickly figure out the values in the problem. It's called the Ratio Box. Let's try the same question, but with some different numbers; if you have 24 coins in your pocket and the ratio of pennies to nickels is 2 : 1, how many pennies and nickels are there? The Ratio Box for this question is below, with all of the information we're given already filled in.

	Pennies	Nickels	Total
ratio	2	1	3
multiply by			
real			24

Remember that ratios are relationships between numbers, not real numbers, so the real total is 24; meaning we have 24 actual coins in our pocket. The ratio total (the number you get when you add up the number of parts in the ratio) is 3.

The middle row of the table is for our multiplier. How do you get from 3 to 24? You multiply by 8. Remember when we talked about finding equivalent fractions? All we did was multiply the numerator and denominator by the same value. That's exactly what we're going to do with ratios. This is what the ratio box would look like now:

	Pennies	Nickels	Total
ratio	2	1	**3**
multiply by	8	8	**8**
real			**24**

Now let's finish filling in the box by multiplying out everything else.

	Pennies	Nickels	Total
ratio	2	1	**3**
multiply by	8	8	**8**
real	16	8	**24**

Let's try a GRE example.

8 of 20

Flour, eggs, yeast, and salt are mixed by weight in the ratio of 11 : 9 : 3 : 2, respectively. How many pounds of yeast are there in 20 pounds of the mixture?

- ○ $1\frac{3}{5}$
- ○ $1\frac{4}{5}$
- ○ 2
- ○ $2\frac{2}{5}$
- ○ $8\frac{4}{5}$

Here's How to Crack It

The minute you see the word "ratio," draw your box on your scratch paper and fill in what you know.

	Flour	Eggs	Yeast	Salt	Total
ratio	11	9	3	2	
multiply by					
real					**20**

First, add up all of the numbers in the ratio to get the ratio total.

	Flour	Eggs	Yeast	Salt	Total
ratio	11	9	3	2	**25**
multiply by					
real					**20**

Now, what do we multiply 25 by to get 20?

$$25x = 20$$

$$\frac{25x}{25} = \frac{20}{25}$$

$$x = \frac{20}{25}$$

$$x = \frac{4}{5}$$

So $\frac{4}{5}$ is our "multiply by" number. Let's fill it in.

	Flour	Eggs	Yeast	Salt	Total
ratio	11	9	3	2	**25**
multiply by	$\frac{4}{5}$	$\frac{4}{5}$	$\frac{4}{5}$	$\frac{4}{5}$	$\frac{4}{5}$
real					**20**

The question asks for the amount of yeast, so we don't have to worry about the other ingredients, just look at the yeast column. All we have to do is multiply 3 by $\frac{4}{5}$ and we have our answer: $3 \times \frac{4}{5} = \frac{12}{5}$, or $3 \times \frac{4}{5} = \frac{12}{5}$, which is answer choice (D).

What Is a Proportion?

So you know that a fraction is a relationship between part and whole, and that a ratio is a relationship between part and part. A proportion is an equivalent relationship between two fractions or ratios. Thus, $\frac{1}{2}$ and $\frac{4}{8}$ are proportionate because they are equivalent fractions. But $\frac{1}{2}$ and $\frac{2}{3}$ are not in proportion because they are not equal ratios.

The GRE often contains problems in which you are given two proportional, or equal, ratios from which one piece of information is missing. These questions take a relationship or ratio, and project it onto a larger or smaller scale. Proportion problems are recognizable because they always give you three values and ask for a fourth value. Here's an example:

The key to proportions is setting them up correctly.

10 of 20

If the cost of a one-hour telephone call is $7.20, what would be the cost in dollars of a ten-minute telephone call at the same rate?

[ ] dollars

Enter your answer in the box provided.

Here's How to Crack It

It's very important to set up proportion problems correctly. That means using your hand and parking your information on your scratch paper. Be essentially careful to label *everything*. It takes only an extra two or three seconds, but doing this will help you catch lots of errors.

For this question, let's express the ratios as dollars over minutes, because we're being asked to find the cost of a ten-minute call. That means that we have to convert 1 hour to 60 minutes (otherwise it wouldn't be a proportion).

$$\frac{\$}{\text{min}} = \frac{\$7.20}{60} = \frac{x}{10}$$

Now cross-multiply.

$$60x = (7.2)(10)$$
$$60x = 72$$
$$\frac{60x}{60} = \frac{72}{60}$$
$$x = \frac{6}{5}$$

Now we can enter 1.20 into the box.

Relationship Review

You may have noticed a trend in the preceding pages. Each of the major topics covered—fractions, percents, ratios, and proportions—described a particular relationship between numbers. To review:

- A fraction expresses the relationship between a part and the whole.
- A percent is a special type of fraction, one that expresses the relationship of part to whole as a fraction with the number 100 in the denominator.
- A ratio expresses the relationship between part and part. Adding up the parts of a ratio give you the whole.
- A proportion expresses the relationship between equal fractions, percents, or ratios.
- Each of these relationships shares all the characteristics of a fraction. You can reduce them, expand them, multiply them, and divide them using the exact same rules you used for working with fractions.

AVERAGES

The average (arithmetic mean) of a set of numbers is the sum, or total value, of all the numbers in the set divided by the number of numbers in the set. The average of the set {1, 2, 3, 4, 5} is equal to the total of the numbers (1 + 2 + 3 + 4 + 5, or 15) divided by the number of numbers in the set (which is 5). Dividing 15 by 5 gives us 3, so 3 is the average of the set.

GRE average problems always give you two of the three numbers needed.

ETS always refers to an average as an "average (arithmetic mean)." This confusing parenthetical remark is meant to keep you from being confused by other kinds of averages, such as medians and modes. You'll be less confused if you simply ignore the parenthetical remark and know that average means total of the elements divided by the number of elements. We'll tell you about medians and modes later.

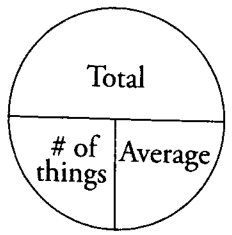
Think Total

Don't try to solve average problems all at once. Do them piece by piece. The key formula to keep in mind when doing problems that involve averages is:

The minute you see the word "average," draw an average pie on your scratch paper.

$$\text{Average} = \frac{\text{Total}}{\text{\# of things}}$$

Drawing an Average Pie will help you organize your information.



Here's how the Average Pie works. The *total* is the sum of the numbers being averaged. The *number of things* is the number of different elements that you are averaging. And the *average* is, naturally, the average.

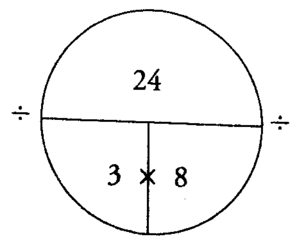
Say you wanted to find the average of 4, 7, and 13. You would add those numbers to get the total and divide that total by three.

$$4 + 7 + 13 = 24$$

$$\frac{24}{3} = 8$$

Which two pieces of the pie do you have?

Mathematically, the Average Pie works like this:



The horizontal bar is a division bar. If you divide the *total* by the *number of things*, you get the *average*. If you divide the total by the *average*, you get the *number of things*. If you have the *number of things* and the *average*, you can simply multiply them together to find the *total*. This is one of the most important things you need to be able to do to solve GRE average problems.

Using the Average Pie has several benefits. First, it's an easy way to organize information. Furthermore, the Average Pie makes it clear that if you have two of the three pieces, you can always find the third. This makes it easier to figure out how to approach the problem. If you fill in the number of things, for example, and the question wants to know the average, the Average Pie shows you that the key to unlocking that problem is finding the total.

Try this one.

15 of 20

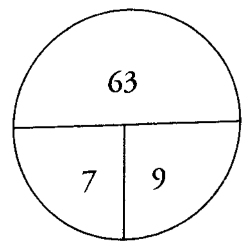
The average of seven numbers is 9. The average of three of these numbers is 5. What is the average of the other four numbers?

- ○ 4
- ○ 5
- ○ 7
- ○ 10
- ○ 12

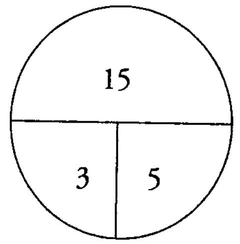
If you see the word "average" twice in a problem, draw two pies. If you see it three times, then draw three pies.

Here's How to Crack It

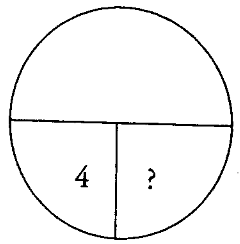
Let's take the first sentence. You have the word "average," so draw your pie and fill in what you know. We have seven numbers with an average of 9, so plug those values into your Average Pie and multiply to find the total.



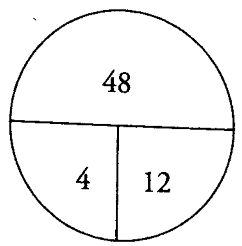
Now we also know that three of the numbers have an average of 5, so draw another Average Pie, plug those values into their places, and multiply to find the total of those three numbers.



The question is asking for the average of the four remaining numbers. Draw one more Average Pie and Plug In 4 for the number of things.



In order to solve for the average, we need to know the total of those four numbers. How do we find this? From our first Average Pie we know that the total of all seven numbers is 63. The second Average Pie tells us that the total of three of those numbers was 15. Thus, the total of the remaining four has to be 63 – 15, which is 48. Plug 48 into the last Average Pie, and divide by 4 to get the average of the four numbers.



The average is 12, which is answer choice (E).

Let's try one more.

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The average (arithmetic mean) of a set of 6 numbers is 28. If a certain number, y, is removed from the set, the average of the remaining numbers in the set is 24.

<u>Quantity A</u>	<u>Quantity B</u>
y	48

○ Quantity A is greater.

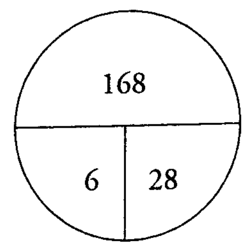
○ Quantity B is greater.

○ The two quantities are equal.

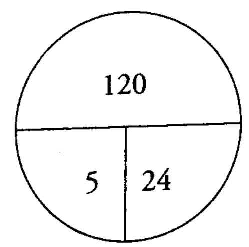
○ The relationship cannot be determined from the information given.

Here's How to Crack It

All right, let's attack this one. The problem says that the "average" of a set of six numbers is 28, so let's immediately draw an average pie and calculate the total.



If a certain number, y, is removed from the set, there are now five numbers left. We already know that the new average is 24, so draw another Average Pie.



The difference between the totals must be equal to y. 168 – 120 = 48. Thus, the two quantities are equal, and the answer is (C).

Up and Down

Averages are very predictable. You should make sure you automatically know what happens to them in certain situations. For example, suppose you take three tests and earn an average score of 90. Now you take a fourth test. What do you know?

If your average goes up as a result of the fourth score, then you know that your fourth score was higher than 90. If your average stays the same as a result of the fourth score, then you know that your fourth score was exactly 90. If your average goes down as a result of the fourth score, then you know that your fourth score was less than 90.

MEDIAN, MODE, AND RANGE

Don't confuse median and mode!

The **median** is the middle value in a set of numbers; above and below the median lie an equal number of values. For example, in the set {1, 2, 3, 4, 5, 6, 7} the median is 4, because it's the middle number (and there are an odd number of numbers in the set). If the set contained an even number of integers {1, 2, 3, 4, 5, 6}, the median would be the average of 3 and 4, or 3.5. When looking for the median, sometimes you have to put the numbers in order yourself. What is the median of the set {13, 5, 6, 3, 19, 14, 8}? First, put the numbers in order from least to greatest, {3, 5, 6, 8, 13, 14, 19}. Then take the middle number. The median is 8. Just think *median = middle* and always make sure the numbers are in order.

The minute you see the word "median" in a question, find a bunch of numbers and put them in order.

The **mode** is the number in a set that occurs most frequently. For example, in the set {2, 3, 4, 5, 3, 8, 6, 9, 3, 9, 3} the mode is 3, because 3 shows up the most. Just think *mode = most.*

The **range** is the difference between the biggest and the smallest numbers in your set. So, in the set {2, 6, 13, 3, 15, 4, 9}, the range is 15 (the highest number in the set) – 2 (the lowest number in the set), or 13.

Here's an example:

12 of 20

What do we need to do to the numbers in this list?

$F = \{4, 2, 7, 11, 8, 9\}$

Quantity A	Quantity B
The range of Set F	The median of Set F

- ○ Quantity A is greater.
- ○ Quantity B is greater.
- ○ The two quantities are equal.
- ○ The relationship cannot be determined from the information given.

Here's How to Crack It

Let's put the numbers in order first, so it'll be easier to see what we have: {2, 4, 7, 8, 9, 11}. First let's look at Quantity A: The range is the largest number, or 11, minus the smallest number, or 2. That's 9. Now let's look at Quantity B: The minute you see the word "median," find a bunch of numbers and put them in order. The median is the middle number of the set, but because there are two middle numbers, 7 and 8, we have to find the average. Or do we? Isn't the average of 7 and 8 clearly going to be smaller than the number in Quantity A, which is 9? Yes (remember, in quant comp questions, we compare, not calculate). The answer is (A).

STANDARD DEVIATION

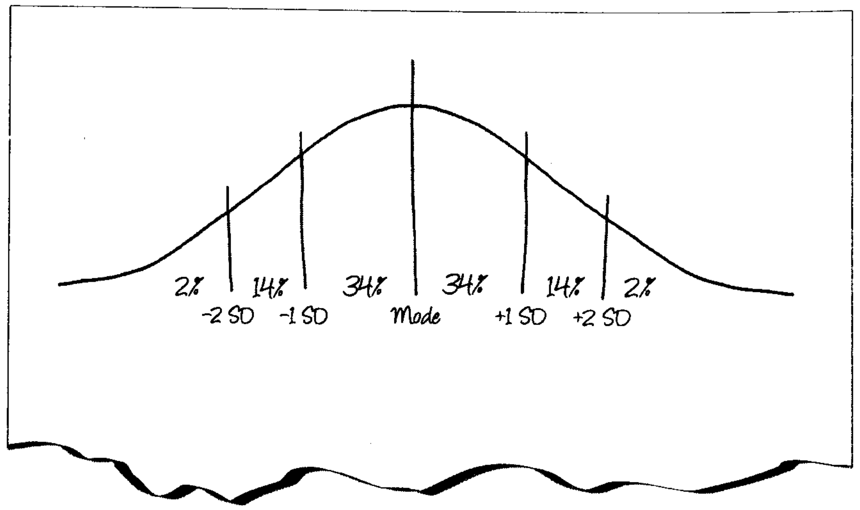
You'll never have to calculate the standard deviation on the GRE.

Standard deviation is one of those phrases that math people like to throw around to scare non-math people, but it's really not that scary. The GRE might ask you questions about standard deviation, but you'll never have to actually calculate it; instead, you'll just need a basic understanding of what standard deviation is. In order to understand standard deviation, we must first look at something all standardized testers should be familiar with, the bell curve.

Your Friend the Bell Curve

The first thing to know about a bell curve is that the number in the middle is the mean, the median, and the mode.

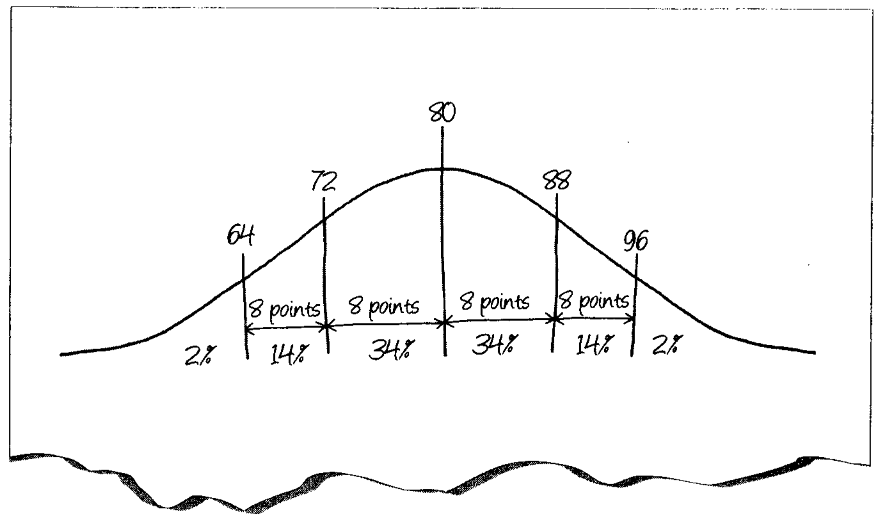
The minute you see the phrase "standard deviation" or "normal distribution," draw your curve and fill in your percentages.



Imagine that 100 students take a test and the results follow a normal distribution. The minute you see the phrase, "normal distribution," draw your curve. Let's say that the average score on this test is an 80. That means that the median and the mode must also be 80. Put 80 in the middle of your curve. You know, however, that a few of those students were extremely well prepared and got a really high score, let's say that 2% of them got a 96 or higher. Put a 96 above the right 2% line on your curve.

Standard deviation measures how much a score differs from the norm (the average) in even increments. The curve tells us that a score earned by only 2% of the students is two standard deviations from the norm. If the norm is 80 and 96 is two standard deviations away; then one standard deviation on this test is 8 points. Two standard deviations above the norm is 96 while two standard deviations below the norm is 64. One standard deviation above the norm is 88, and one standard deviation below the norm is 72. Fill these in on your bell curve.

Now you know quite a bit about the distribution of scores on this test. Sixty-eight percent of the students received a score between 72 and 88. Ninety-eight percent scored above a 64. That's all there is to know about standard deviations. The percentages don't change, so memorize those. When you see the phrase, just make your curve and fill in what you know. Here's what the curve would look like for this test:



When it comes to standard deviation, the percentages don't change, so memorize those: 2, 14, and 34.

Here's an example of how ETS might test standard deviation:

5 of 20

Quantity A	Quantity B
The standard deviation of a set of data consisting of 10 integers ranging from –20 to –5	The standard deviation of a set of data consisting of 10 integers ranging from 5 to 20

- ○ Quantity A is greater.
- ○ Quantity B is greater.
- ○ The two quantities are equal.
- ○ The relationship cannot be determined from the information given.

Here's How to Crack It

ETS is hoping you'll make a couple of wrong turns on this problem. The first trap they set is that one set of numbers contains negative integers while the other doesn't—but this doesn't mean that one set will have a negative standard deviation. Standard deviation is defined as the distance a point is from the mean, so it can never be negative. The second trap is that ETS hopes you'll waste a lot of time trying to calculate standard deviation based on the information given. But you know better than to try to do that. Remember that ETS won't ask you to calculate standard deviation; it's a complex calculation. Plus, as you know, you need to know the mean in order to figure the standard deviation and there's no way we can find it based on the information here. Thus, we have no way of comparing these two quantities, and our answer is (D).

Now let's try a question that will make use of the bell curve.

14 of 20

The fourth grade at School x is made up of 300 students who have a total weight of 21,600 pounds. If the weight of these fourth graders has a normal distribution and the standard deviation equals 12 pounds, approximately what percentage of the fourth graders weighs more than 84 pounds?

- ○ 12%
- ○ 16%
- ○ 36%
- ○ 48%
- ○ 60%

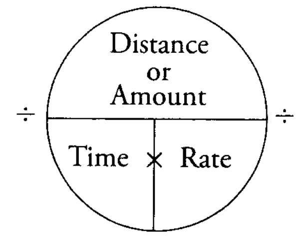
Here's How to Crack It

This one's a little tougher than the earlier standard deviation questions. The first step is to determine the average weight of the students, which is $\frac{21,600}{300} = 72$ pounds. If the standard deviation is 12 pounds, then 84 pounds places us exactly one standard deviation above the mean, or at the 84th percentile (remember the bell curve?). Because 16 percent of all students weigh more than 84 pounds, the answer is (B).

RATE

Rate problems are similar to average problems. A rate problem might ask for an average speed, distance, or the length of a trip, or how long a trip (or a job) takes. To solve rate problems, use the Rate Pie.

A rate problem is really just an average problem.



The Rate Pie works exactly the same way as the Average Pie. If you divide the *distance* or *amount* by the *rate,* you get the *time.* If you divide the *distance* or *amount* by the *time,* you get the *rate.* If you multiply the *rate* by the *time,* you get the *distance* or *amount.*

Let's take a look.

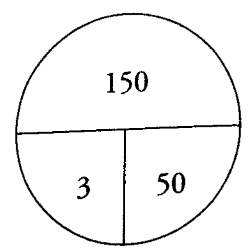
16 of 20

It takes Carla three hours to drive to her brother's house at an average speed of 50 miles per hour. If she takes the same route home, but her average speed is 60 miles per hour, how long does it take her to get home?

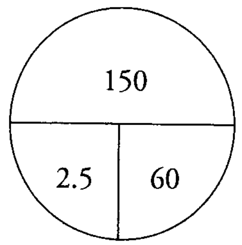
- ○ 2 hours
- ○ 2 hours and 14 minutes
- ○ 2 hours and 30 minutes
- ○ 2 hours and 45 minutes
- ○ 3 hours

Here's How to Crack It

The trip to her brother's house takes three hours, and the rate is 50 miles per hour. Plug those numbers into a Rate Pie and multiply to find the distance.



So the distance is 150 miles. On her trip home, Carla travels at a rate of 60 miles per hour. Draw another Rate Pie and Plug In 150 and 60. Then all you have to do is divide 150 by 60 to find the time.



So it takes Carla two and a half hours to get home. That's answer choice (C).

Try another one.

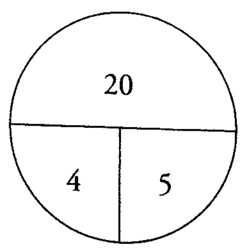
15 of 20

A machine can stamp 20 envelopes in 4 minutes. How many of these machines, working simultaneously, are needed to stamp 60 envelopes per minute?

- ○ 5
- ○ 10
- ○ 12
- ○ 20
- ○ 24

Here's How to Crack It

First we have to find the rate per minute of one machine. Plug 20 and 4 into a Rate Pie and divide to find the rate.



The rate is 5. If one machine can stamp 5 envelopes per minute, how many machines do you need to stamp 60 per minute? $60 \div 5 = 12$, or answer choice (C).

CHARTS

Every GRE Math section has a few questions that are based on a chart or graph (or on a group of charts or graphs). But don't worry; the most important thing that chart questions test is your ability to remember the difference between real-life charts and ETS charts.

Chart questions frequently test percents, percent change, ratios, proportions, and averages.

In real life, charts are often provided in order to display information in a way that's easier to understand. Conversely, ETS constructs charts to hide information you need to know and to make that information harder to understand.

Chart Questions

There are usually two or three questions per chart or per set of charts. Like the reading comprehension questions, chart questions appear on split screens. Be sure to click on the scroll bar and scroll down as far as you can; there may be additional charts underneath the top one, and you want to make sure you've seen all of them.

Chart problems just recycle the basic arithmetic concepts we've already covered: fractions, percentages, and so on. This means you can use the techniques we've discussed for each type of question, but there are two additional techniques that are especially important to use when doing chart questions.

On charts, look for the information ETS is trying to hide.

Don't Start with the Questions: Start with the Charts

Take a minute to note the following key bits of information from any chart you see.

- **Information in titles:** Make sure you know what each chart is telling you.
- **Asterisks, footnotes, parentheses, and small print:** Often there will be crucial information hidden away at the bottom of the chart. Don't miss it!
- **Funny units:** Pay special attention when a title says "in thousands" or "in millions." You can usually ignore the units as you do the calculations, but you have to use them to get the right answer.

Don't try to work with huge values. Ballpark instead!

Approximate, Estimate, and Ballpark

Like some of our other techniques, you have to train yourself to estimate when looking at charts and graphs. You should estimate, not calculate exactly:

- Whenever you see the word *approximately* in a question
- Whenever the question has answer choices and when the answer choices are far apart in value
- Whenever you start to answer a question and you justifiably say to yourself, "This is going to take a lot of calculation!"

Review those "friendly" percentages and their fractions from earlier in the chapter. Try estimating this question:

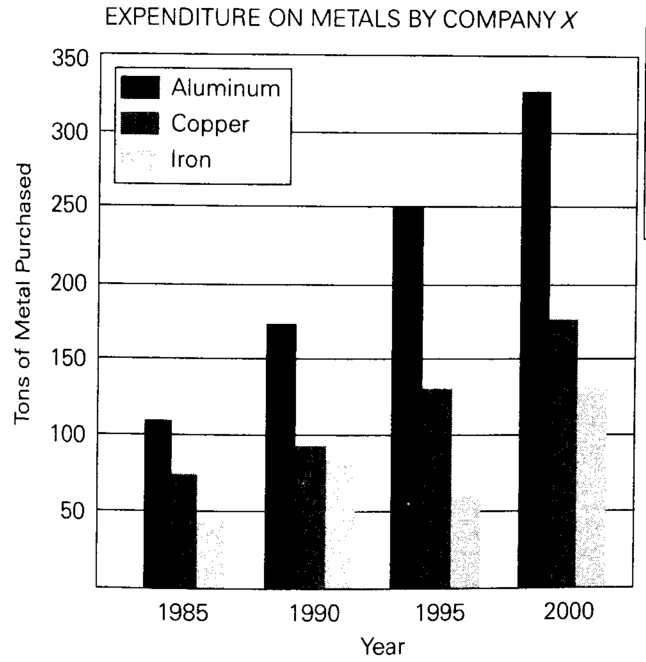
What is approximately 9.6 percent of 21.4?

Here's How to Crack It

Use 10 percent as a friendlier percentage and 20 as a friendlier number. One-tenth of 20 is 2 (it says "approximately"—who are you to argue?). That's all you need to do to answer most chart questions.

Chart Problems

Make sure you've read everything on the chart carefully before you try the first question.



Year	Price of Aluminum Per Ton
1985	$1,900
1990	$2,200
1995	$2,700
2000	$3,400

Note: Graphs drawn to scale.

6 of 20

Approximately how many tons of aluminum and copper combined were purchased in 1995 ?

○ 125

○ 255

○ 325

○ 375

○ 515

7 of 20

How much did Company X spend on aluminum in 1990 ?

- $675,000
- $385,000
- $333,000
- $165,000
- $139,000

8 of 20

Approximately what was the percent increase in the price of aluminum from 1985 to 1995 ?

- 8%
- 16%
- 23%
- 30%
- 42%

Here's How to Crack the First Question

As you can see from the graph on the previous page, in 1995, the black bar (which indicates aluminum) is at 250, and the dark grey bar (which indicates copper) is at approximately 125. Add those up and you get the number of tons of aluminum and copper combined that were purchased in 1995: 250 + 125 = 375. That's choice (D). Notice that the question says "approximately." Also notice that the numbers in the answer choices are pretty far apart.

Here's How to Crack the Second Question

We need to use the chart and the graph to answer this question, because we need to find the number of tons of aluminum purchased in 1990 and multiply it by the price per ton of aluminum in 1990 in order to figure out how much was spent on aluminum in 1990. The bar graph tells us that 175 tons of aluminum was purchased in 1990, and the little chart tells us that aluminum was $2,200 per ton in 1990. 175 × $2,200 = $385,000. That's choice (B).

Here's How to Crack the Third Question

Remember that percent increase formula from earlier in this chapter?

$$\text{Percent change} = \frac{\text{Difference}}{\text{Original}} \times 100$$

We'll need to use the little chart for this one. In 1985, the price of aluminum was $1,900 per ton. In 1995, the price of aluminum was $2,700 per ton. Now let's use the formula. 2,700 – 1,900 = 800, so that's the difference. This is a percent increase problem, so the original number is the smaller one. Thus, the original is 1,900, and our formula looks like this: Percent change = $\frac{800}{1,900} \times 100$. By canceling the 0's in the fraction you get $\frac{8}{19} \times 100$, and multiplying gives you $\frac{800}{19}$. At this point you could divide 800 by 19 to get the exact answer, but because they're looking for an approximation, let's round 19 to 20. What's 800 ÷ 20? That's 40, and answer choice (E) is the only one that's close.

Real World Math Drill

Now it's time to try out what you've learned on some practice questions. Try the following problems and then check your answers in Part V.

1 of 19

If $3(r + s) = 7$, then, in terms of r, $s =$

- ○ $\frac{7}{3} - r$
- ○ $\frac{7}{3} + r$
- ○ $7 - 3r$
- ○ $\frac{7}{3} - \frac{r}{3}$
- ○ $\frac{7}{3} + \frac{r}{3}$

2 of 19

Sadie distributes her collection of paintings, giving one-third of the paintings to friends and selling half of the paintings. If she keeps the remaining paintings, what fraction of her collection does Sadie keep?

☐

Click on the answer box, then type in a number. Backspace to erase.

3 of 19

During a sale, a store decreases prices on all its scarves by 25 to 50 percent. If all of the scarves in the store originally cost \$20, which of the following could be the sale price of a scarf?

*Indicate **all** possible values.*

- ☐ \$8
- ☐ \$10
- ☐ \$12
- ☐ \$14
- ☐ \$16

Click on your choice(s).

4 of 19

Quantity A	Quantity B
12 percent of 35	35 percent of 12

- ○ Quantity A is greater.
- ○ Quantity B is greater.
- ○ The two quantities are equal.
- ○ The relationship cannot be determined from the information given.

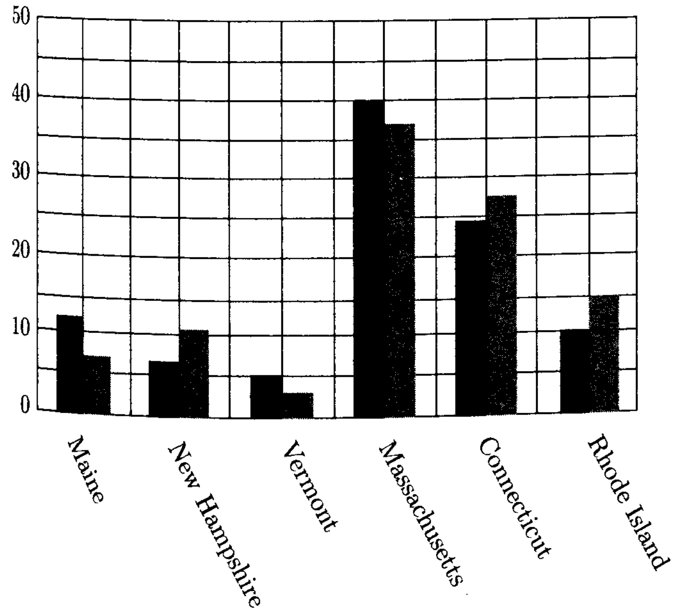
5 of 19

Quantity A	Quantity B
$\frac{2.6}{0.259}$	10

- ○ Quantity A is greater.
- ○ Quantity B is greater.
- ○ The two quantities are equal.
- ○ The relationship cannot be determined from the information given.

Questions 6 through 9 refer to the following graph.

PERCENT OF POPULATION IN NEW ENGLAND BY STATE IN YEAR X AND YEAR Y



Note: Drawn to scale

■ Year X: Total New England population= 15 million

■ Year Y: Total New England population= 25 million

6 of 19

The six New England states are ranked by population in Year X and in Year Y. How many states had a different ranking from Year X to Year Y?

- ○ None
- ○ One
- ○ Two
- ○ Three
- ○ Four

7 of 19

In Year Y, the population of Massachusetts was approximately what percent of the population of Vermont?

- ○ 50%
- ○ 120%
- ○ 300%
- ○ 400%
- ○ 1,200%

8 of 19

By approximately how much did the population of Rhode Island increase from Year X to Year Y?

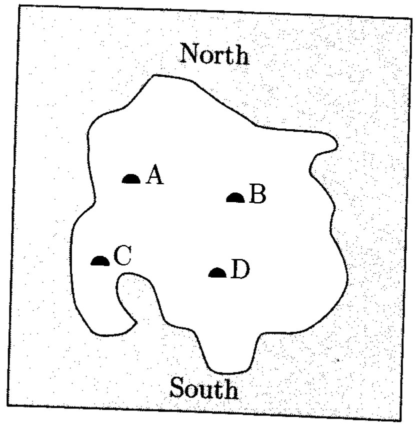
- ○ 750,000
- ○ 1,250,000
- ○ 1,500,000
- ○ 2,250,000
- ○ 3,375,000

9 of 19

Approximately what is the difference between the percent change of Connecticut's percent of total New England population from Year X to Year Y and the percent change of Massachusetts percent of total New England population from Year X to Year Y ?

- ○ -12.5
- ○ 5
- ○ 12.5
- ○ 25
- ○ 50

10 of 19



Towns A, B, C, and D are located on the map as shown. Towns A and B have 3,000 people each who support referendum R, and the referendum has an average of 3,500 supporters in towns B and D.

Quantity A	**Quantity B**
The average number of supporters of referendum R in the two southern-most towns	2,500

- ○ Quantity A is greater.
- ○ Quantity B is greater.
- ○ The two quantities are equal.
- ○ The relationship cannot be determined from the information given.

11 of 19

A company paid \$500,000 in merit raises to employees whose performances were rated A, B, or C. Each employee rated A received twice the amount of the raise that was paid to each employee rated C; each employee rated B received one-and-a-half times the amount of the raise that was paid to each employee rated C. If 50 workers were rated A, 100 were rated B, and 150 were rated C, how much was the raise paid to each employee rated A ?

- ○ \$370
- ○ \$625
- ○ \$740
- ○ \$1,250
- ○ \$2,500

12 of 19

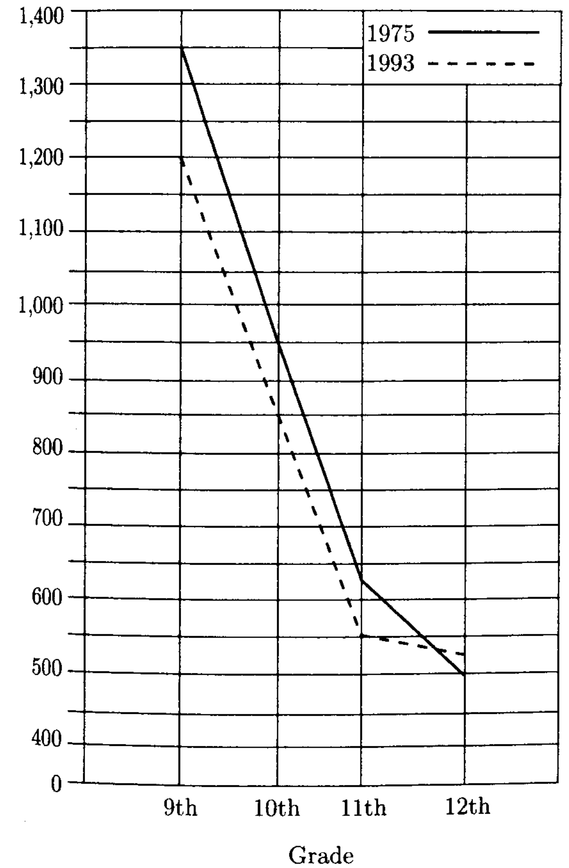
The original price of an item at a store is 40 percent more than the price the retailer paid for it. To encourage sales, the retailer reduces the price of the item by 15 percent from the original selling price. If the retailer sells the item at the reduced cost, his profit is what percent of his cost?

[ ] percent

Click on the answer box, then type in a number. Backspace to erase.

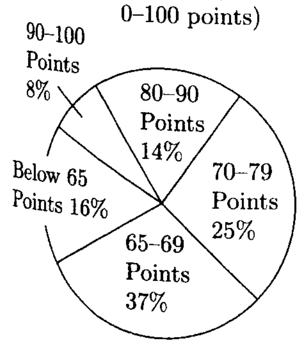
Questions 13 through 15 refer to the following graphs.

NUMBER OF STUDENTS IN GRADES 9 THROUGH 12 FOR SCHOOL DISTRICT X IN 1975 AND 1993

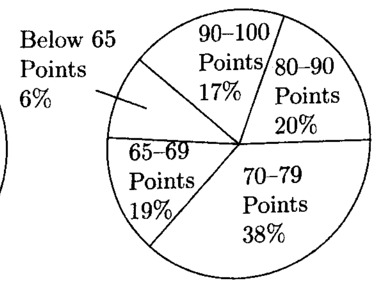


DISTRIBUTION OF READING TEST SCORES* FOR SCHOOL DISTRICT X STUDENTS IN 1993

(*Reading Test scores can range from 0–100 points)



9th Grade Students



10th–12th Grade Students

Note: Drawn to scale.

13 of 19

In 1993, the median reading test score for ninth grade students was in which score range?

- ○ Below 65 points
- ○ 65–69 points
- ○ 70–79 points
- ○ 80–89 points
- ○ 90–100 points

14 of 19

If the number of students in grades 9 through 12 comprised 35 percent of the number of students in School District X in 1975, then approximately how many students were in School District X in 1975 ?

- ○ 9,700
- ○ 8,700
- ○ 3,400
- ○ 3,000
- ○ 1,200

15 of 19

Assume that all students in School District X took the reading test each year. In 1993, approximately how many more ninth grade students had reading test scores in the 70–79 point range than in the 80–89 point range?

- ○ 470
- ○ 300
- ○ 240
- ○ 170
- ○ 130

16 of 19

Quantity A	Quantity B
$2 - \frac{27}{25}$	$\frac{3}{5} + \frac{12}{125}$

- ○ Quantity A is greater.
- ○ Quantity B is greater.
- ○ The two quantities are equal.
- ○ The relationship cannot be determined from the information given.

17 of 19

Solution X contains only ingredients a and b in a ratio of 2 : 3. Solution Y contains only ingredients a and b in a ratio of 1 : 2. If Solution Z is created by mixing solutions X and Y in a ratio of 3 : 11, then 630 ounces of Solution Z contains how many ounces of a ?

- ○ 68
- ○ 73
- ○ 89
- ○ 219
- ○ 236

18 of 19

On Sunday, Belmond Public Library has 160 books, none of which have been checked out. On Monday, 40 of the books are checked out. On Tuesday, $\frac{1}{2}$ of the borrowed books are returned. Wednesday, $\frac{1}{2}$ of the books still checked out are returned and then 20 more are checked out. On Thursday, a wealthy patron donates 80 books, and $\frac{1}{6}$ of the books still checked out are returned. On Friday 30 more books are borrowed, and on Saturday 35 are checked out. What is the percent change from the books in the library at the end of the day on Monday to the books in the library at end of the day the following Saturday?

[ ] percent

Click on the answer box, then type in a number. Backspace to erase.

19 of 19

Jill has received 8 of her 12 evaluation scores. So far, Jill's average (arithmetic mean) is 3.75 out of a possible 5. If Jill needs an average of 4.0 points to get a promotion, which set of scores will allow Jill to receive her promotion?

*Indicate **all** possible answers.*

- ☐ 3.0, 3.5, 4.75, 4.75
- ☐ 3.5, 4.75, 4.75, 5.0
- ☐ 3.25, 4.5, 4.75, 5.0
- ☐ 3.75, 4.5, 4.75, 5.0

Click on your choice(s).

Summary

- Fractions, decimals, and percents are all ways of expressing parts of integers.
- Translation is a useful tool for converting fraction and percent problems into mathematical equations.
- Percent change is expressed as the difference between two numbers divided by the original number.
- Plug In on questions that ask about percents or fractions of an unknown amount.
- A ratio expresses a part to part relationship. The key to ratio problems is finding the total. Use the ratio box to organize ratio questions.
- A proportion expresses the relationship between equal fractions, percents, or ratios. A proportion problem always provides you with three pieces of information and asks you for a fourth.
- Use the Average Pie to organize and crack average problems.
- The median is the middle number in a set of values. The mode is the value that appears most frequently in a set. The range of a set is the difference between the largest and smallest value in the set.
- You will never have to calculate standard deviation on the GRE.
- Standard deviation problems are really average and percent problems. Make sure you know the percentages associated with the bell curve: 34%–14%–2%.
- Use the Rate Pie for rate questions.
- On chart questions, make sure you take a moment to understand what information the chart is providing. Estimate answers to chart questions whenever possible.

Chapter 11
Geometry

Chances are you probably haven't used the Pythagorean theorem recently or had to find the area of a circle in quite a while. However, you'll be expected to know geometry concepts such as these on the new GRE. This chapter reviews all the important rules and formulas you'll need to crack the geometry problems on the GRE. It also provides examples of how such concepts will be tested on the GRE Math section.

Expect to see a handful of basic geometry problems on each of your Math sections.

WHY GEOMETRY?

Good question. If you're going to graduate school for political science or linguistics or history or practically anything that doesn't involve math, you might be wondering why the heck you have to know the area of a circle or the Pythagorean theorem for this exam. While we may not be able to give you a satisfactory answer to that question, we can help you do well on the geometry questions on the GRE.

WHAT YOU NEED TO KNOW

The good news is that you don't need to know much about actual geometry to do well on the GRE; we've boiled down geometry to the handful of bits and pieces that ETS actually tests.

Before we begin, consider yourself warned: Since you'll be taking your test on a computer screen, you'll have to be sure to transcribe all the figures onto your scrap paper accurately. All it takes is one mistaken angle or line and you're sure to get the problem wrong. So make ample use of your scratch paper and always double-check your figures. Start practicing now, by using scratch paper with this book.

Problem-solving questions will be drawn to scale unless they clearly tell you otherwise. Quant comp questions, on the other hand, may *not* be drawn to scale, so be on your guard!

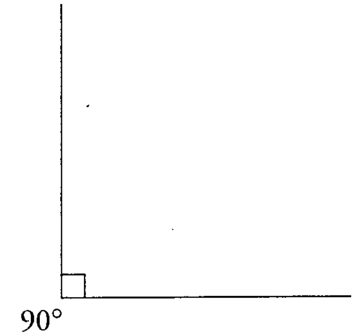
Another important thing to know is that you cannot necessarily trust the diagrams ETS gives you. Sometimes they are very deceptive and are intended to confuse you. Always go by what you read, not what you see.

DEGREES, LINES, AND ANGLES

For the GRE, you will need to know that

1. A line is a 180-degree angle. In other words, a line is a perfectly flat angle.
2. When two lines intersect, four angles are formed; the sum of these angles is 360 degrees.
3. When two lines are perpendicular to each other, their intersection forms four 90-degree angles. Here is the symbol ETS uses to indicate a perpendicular angle: $\perp$.

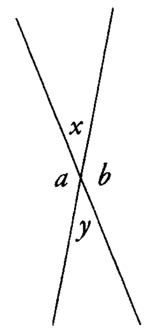
4. Ninety-degree angles are also called *right angles.* A right angle on the GRE is identified by a little box at the intersection of the angle's arms:



5. The three angles inside a triangle add up to 180 degrees.
6. The four angles inside any four-sided figure add up to 360 degrees.
7. A circle contains 360 degrees.
8. Any line that extends from the center of the circle to the edge of the circle is called a *radius* (plural is *radii*).

Vertical Angles

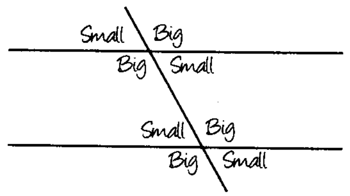
Vertical angles are the angles that are across from each other when two lines intersect. Vertical angles are always equal. In the drawing below, angle x is equal to angle y (they are vertical angles) and angle a is equal to angle b (they are also vertical angles).



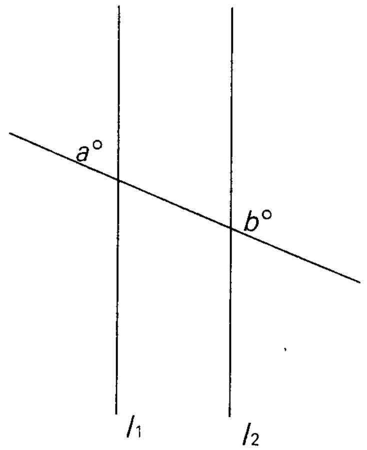
On the GRE, the measure of only one of the vertical angles is typically shown. But usually you'll need to use the other angle to solve the problem.

Parallel Lines

Parallel lines are lines that never intersect. When a pair of parallel lines is intersected by a third, two types of angles are formed: big angles and small angles. Any big angle is equal to any big angle, and any small angle is equal to any small angle. The sum of any big angle and any small angle will always equal 180. When ETS tells you that two lines are parallel, this is what is being tested. The symbol for parallel lines and the word "parallel" are both clues that tell you what to look for in the problem. The minute you see either of them, immediately identify your big and small angles; they will probably come into play.



4 of 20



l_1 and l_2 are parallel

Quantity A	Quantity B
$a + b$	180

- ○ Quantity A is greater.
- ○ Quantity B is greater.
- ○ The two quantities are equal.
- ○ The relationship cannot be determined from the information given.

Here's How to Crack It

Notice that you're told that these lines are parallel. Here's one very important point: You need to be told that. You can't assume that they are parallel just because they look like they are.

Okay, so as you just learned, only two angles are formed when two parallel lines are intersected by a third line: a big angle (greater than 90 degrees) and a small one (smaller than 90 degrees). Look at angle *a*. It looks smaller than 90, right? Now look at angle *b*. It looks bigger than 90, right? You also know that $a + b$ must add up to 180. The answer is (C).

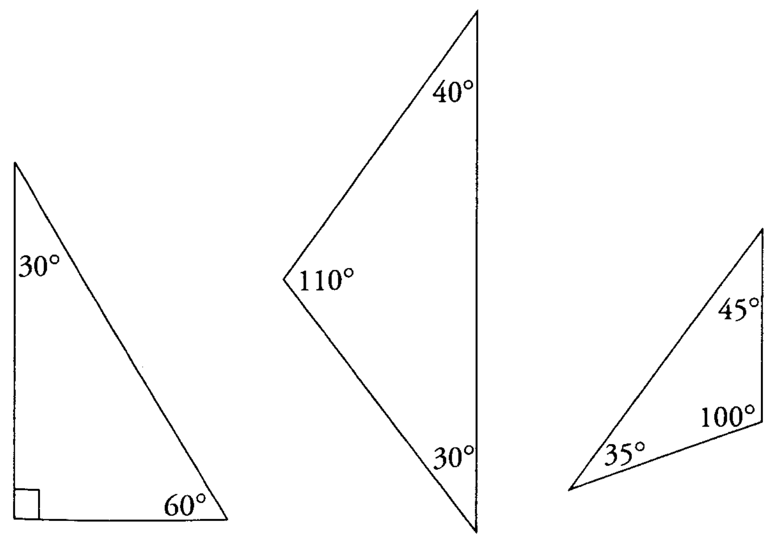
TRIANGLES

Triangles are perhaps ETS's favorite geometrical shape. Triangles have many properties, which make them great candidates for standardized test questions. Make sure you familiarize yourself with the following triangle facts.

Triangles are frequently tested on the GRE.

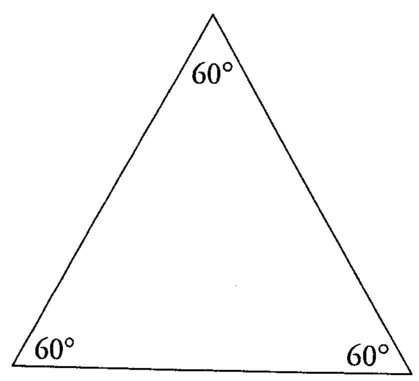
Angles in a Triangle

Every triangle contains three angles that add up to 180 degrees. You must know this fact cold for the exam. This rule applies to every triangle, no matter what it looks like. Here are some examples:



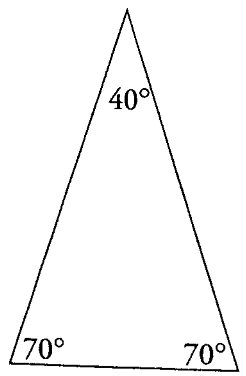
Equilateral Triangles

An equilateral triangle is a triangle in which all three sides are equal in length. Because all of the sides are equal in these triangles, all of the angles are equal. Each angle is 60 degrees because 180 divided by 3 is 60.



Isosceles Triangles

An isosceles triangle is a triangle in which two of the three sides are equal in length. This means that two of the angles are also equal.



Angle/Side Relationships in Triangles

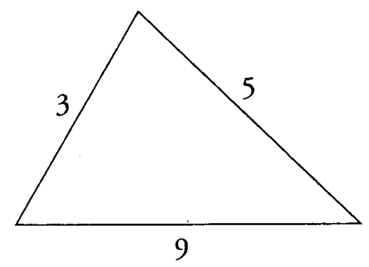
In any triangle, the longest side is opposite the largest interior angle; the shortest side is opposite the smallest interior angle. That's why the hypotenuse of a right triangle is its longest side—there couldn't be another angle in the triangle bigger than 90 degrees. Furthermore, equal sides are opposite equal angles.

Perimeter of a Triangle

The perimeter of a triangle is simply a measure of the distance around it. All you have to do to find the perimeter of a triangle is add up the lengths of the sides.

The Third Side Rule

Why is it impossible for the following triangle to exist? (Hint: It's not drawn to scale.)



This triangle could not exist because the length of any one side of a triangle is limited by the lengths of the other two sides. This can be summarized by the **Third Side rule:**

> The length of any one side of a triangle must be less than the sum of the other two sides and greater than the difference between the other two sides.

This rule is not tested frequently on the GRE, but when it is, it's usually the key to solving the problem. Here's what the rule means in application: Take the lengths of any two sides of a triangle. Add them together, then subtract one from the other. The length of the third side must lie between those two numbers.

Take the sides 3 and 5 from the triangle above. What's the longest the third side could measure? Just add and subtract. It could not be as long as 8 (5 + 3) and it could not be as short as 2 (5 – 3).

Therefore, the third side must lie between 2 and 8. It's important to remember that the third side cannot be equal to either 2 or 8. It must be greater than 2 and less than 8.

Try the following question:

14 of 20

A triangle has sides 4, 7, and x. Which of the following could be the perimeter of the triangle?

*Indicate **all** possible values.*

☐ 13

☐ 16

☐ 17

☐ 20

☐ 22

Any time you see the word "area" or any other word that indicates that a formula is to be used, write the formula on your scratch paper and park the information you're given directly underneath.

Here's How to Crack It

The perimeter of a triangle is equal to the sum of it's three sides. So far, we have sides of 4 and 7, so our partial perimeter is $4 + 7 = 11$. What about the third side, x? The third-side rule tells us that the side could not be longer than $7 + 4 = 11$ or shorter than $7 - 4 = 3$. The third side must be greater than 3 and less than 11. Next we add the partial perimeter, 11, to both of these numbers to find the range of the perimeter. $11 + 3 = 14$ and $11 + 11 = 22$, so the perimeter must be greater than 14 and less than 22. Only choices (A) and (E) fall outside this range. For this question, we have to click on all of the answers that work, so the best answer is (B), (C), and (D).

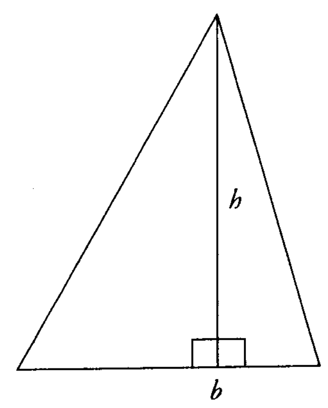
Area of a Triangle

The area of any triangle is equal to its height (or "altitude") multiplied by its base, divided by 2, so

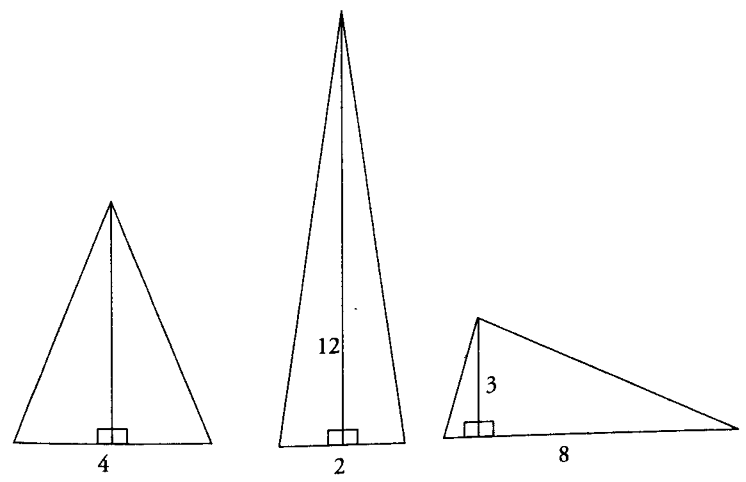
$$A = \frac{1}{2}bh$$

The height of a triangle is defined as the length of a perpendicular line drawn from the point of the triangle to its base.

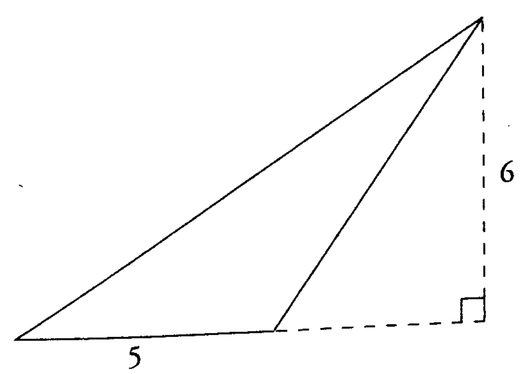
The height of the triangle must be perpendicular to the base.



This area formula works on any triangle.



area = 12   *area* = 12   *area* = 12

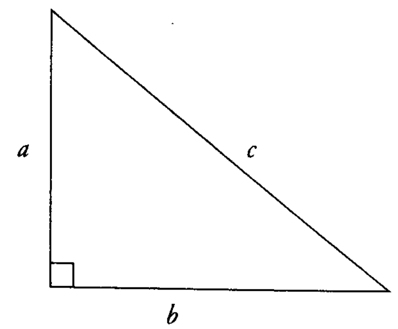


area = 15

ETS will sometimes try to intimidate you by using multiples of the common Pythagorean triples. For example, you might see a 10-24-26 triangle. That's just a 5-12-13 in disguise though.

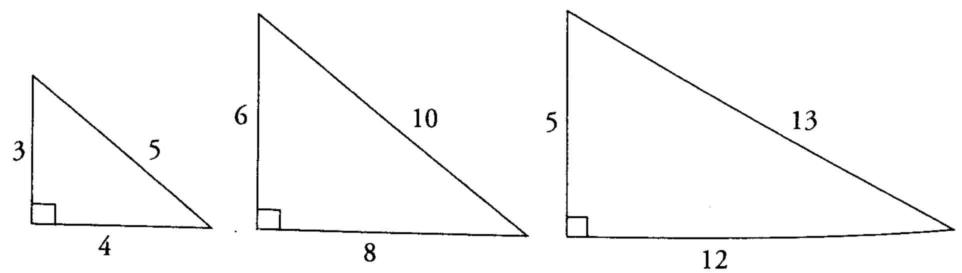
The Pythagorean Theorem

The Pythagorean theorem applies only to right triangles. This theorem states that in a right triangle, the square of the length of the hypotenuse (the longest side, remember?) is equal to the sum of the squares of the lengths of the two other sides. In other words, $c^2 = a^2 + b^2$, where c is the length of the hypotenuse and a and b are the lengths of the other sides. (The two sides that are not the hypotenuse are called the legs.)



You can always calculate the third side of a right triangle using the Pythagorean theorem.

Here are the most common right triangles:



Note that a triangle could have sides with actual lengths of 3, 4, and 5, or 3:4:5 could just be the ratio of the sides. If you double the ratio, you get a triangle with sides equal to 6, 8, and 10. If you triple it, you get a triangle with sides equal to 9, 12, and 15.

Let's try an example.

Write everything down on scratch paper! Don't do anything in your head.

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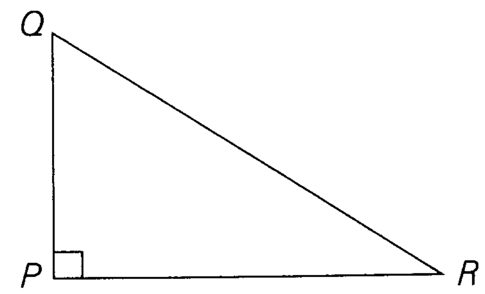

In the figure above, if the distance from point P to point Q is 6 miles and the distance from point Q to point R is 10 miles, what is the distance from point P to point R?

- ○ 4
- ○ 5
- ○ 6
- ○ 7
- ○ 8

Here's How to Crack It

Once you've sensitized yourself to the standard right triangles, this problem couldn't be easier. When you see a right triangle, be suspicious. One leg is 6. The hypotenuse is 10. The triangle has a ratio of 3:4:5. Therefore, the third side (the other leg) must be 8.

The Pythagorean theorem will sometimes help you solve problems that involve squares or rectangles. For example, every rectangle or square can be divided into two right triangles. This means that if you know the length and width of any rectangle or square, you also know the length of the diagonal—it's the shared hypotenuse of the hidden right triangles.

Here's an example:

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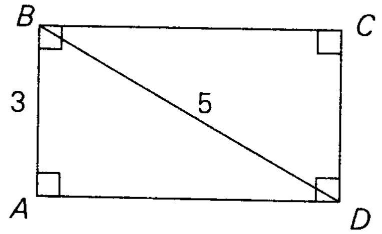


In the rectangle above, what is the area of triangle ABD ?

Enter your answer in the box above.

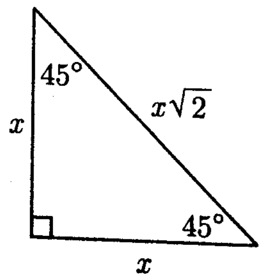
Here's How to Crack It

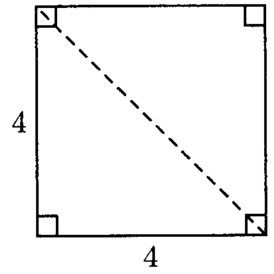
We were told that this is a rectangle (remember that you can never assume!), which means that triangle ABD is a right triangle. Not only that, but it's a 3-4-5 right triangle (with a side of 3 and a hypotenuse of 5, it must be), with side $AD = 4$. So, the area of triangle ABD is $\frac{1}{2}$ the base (3) times the height (4). That's $\frac{1}{2}$ of 12, otherwise known as 6. You could enter that value into the box.

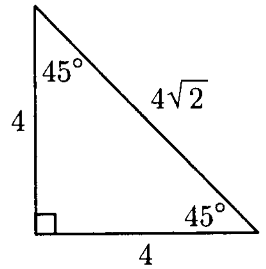
Right Isosceles Triangles

You always know the length of the diagonal of a square because it is one side of the square times $\sqrt{2}$.

If you take a square and cut it in half along its diagonal, you will create a right isosceles triangle. The two sides of the square stay the same. The 90-degree angle will stay the same, and the other two angles that were 90 degrees each get cut in half and are now 45 degrees. The ratio of sides in a right isosceles triangle is $x : x : x\sqrt{2}$. This is significant for two reasons. First, if you see a problem with a right triangle and there is a $\sqrt{2}$ anywhere in the problem, you know what to look for. Second, you always know the length of the diagonal of a square because it is one side times the square root of two.







Let's try an example involving a special right triangle.

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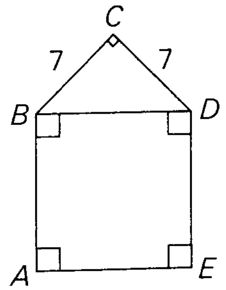


In the figure above, what is the area of square $ABDE$?

- ○ $28\sqrt{2}$
- ○ 49
- ○ $49\sqrt{2}$
- ○ 98
- ○ $98\sqrt{2}$

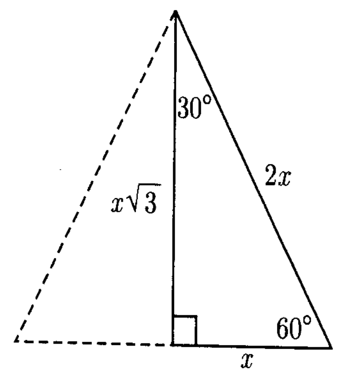
Here's How to Crack It

In order to figure out the area of square $ABDE$, we need to know the length of one of its sides. We can get the length of BD by using the isosceles right triangle attached to it. BD is the hypotenuse, which means its length is $7\sqrt{2}$. To get the area of the square we have to square the length of the side we know, or $(7\sqrt{2})(7\sqrt{2}) = (49)(2) = 98$. That's choice (D).

You can always calculate the area of an equilateral triangle because you know that the height is one half of one side times $\sqrt{3}$.

30 : 60 : 90 Triangles

If you take an equilateral triangle and draw in the height, you end up cutting it in half and creating a right triangle. The hypotenuse of the right triangle has not changed; it's just one side of the equilateral triangle. One of the 60 degree angles stays the same as well. The angle where the height meets the base is 90 degrees, naturally, and the side that was the base of the equilateral triangle has been cut in half. The smallest angle, at the top, opposite the smallest side, is 30 degrees. The ratio of sides on a 30°-60°-90° triangle is $x : x\sqrt{3} : 2x$. Here's what it looks like:



This is significant for two reasons. The first is that if you see a problem with a right triangle and one side is double the other or there is a $\sqrt{3}$ anywhere in the problem, you know what to look for. The second is that you always know the area of an equilateral triangle because you always know the height. It is one half of one side times the square root of three.

Here's one more:

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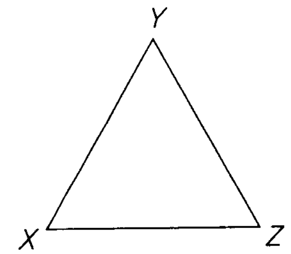


Triangle XYZ in the figure above is an equilateral triangle. If the perimeter of the triangle is 12, what is its area?

○ $2\sqrt{3}$

○ $4\sqrt{3}$

○ 8

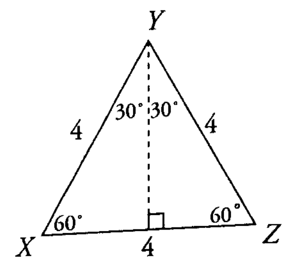
○ 12

○ $8\sqrt{3}$

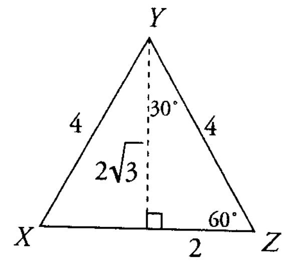
If you see $\sqrt{2}$ or $\sqrt{3}$ in the answer choices of the problem it's a tip-off that the problem is testing special right triangles.

Here's How to Crack It

Here we have an equilateral triangle with a perimeter of 12, which means that each side has a length of 4 and each angle is 60 degrees. Remember that in order to find the area of a triangle, we use the triangle area formula: $A = \frac{1}{2}bh$, but first we need to know the base and the height of the triangle. The base is 4, which now gives us $A = \frac{1}{2}4h$, and now the only thing we need is the height. Remember: The height always has to be perpendicular to the base. Draw a vertical line that splits the equilateral triangle in half. The top angle is also split in half, so now we have this:



What we've done is create two 30°-60°-90° right triangles, and we're going to use one of these right triangles to find the height. Let's use the one on the right. We know that the hypotenuse in a 30°-60°-90° right triangle is always twice the length of the short side. Here we have a hypotenuse (*YZ*) of 4, so our short side has to be 2. The long side of a 30°-60°-90° right triangle is always equal to the short side multiplied by the square root of 3. So if our short side is 2, then our long side must be $2\sqrt{3}$. That's the height.



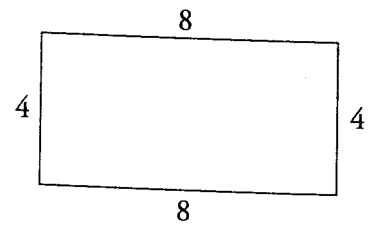
Finally, we return to our area formula. Now we have $A = \frac{1}{2} \times 4 \times 2\sqrt{3}$. Multiply it out and you get $A = 4\sqrt{3}$. The answer is (B).

FOUR-SIDED FIGURES

The four angles inside any figure that has four sides add up to 360 degrees. That includes rectangles, squares, and parallelograms. Parallelograms are four-sided figures made out of two sets of parallel lines whose area can be found with the formula $A = bh$, where h is the height of a line drawn perpendicular to the base.

Perimeter of a Rectangle

The perimeter of a rectangle is just the sum of the lengths of its four sides.



perimeter = 4 + 8 + 4 + 8

The area of a rectangle is equal to its length times its width. For example, the area of the rectangle above is 32 (or 8 × 4).

Area of a Rectangle

The area of a rectangle is equal to its length times its width. For example, the area of the rectangle above is 32 (or 8 × 4).

Squares

A square has four equal sides. The perimeter of a square is, therefore, 4 times the length of any side. The area of a square is equal to the length of any side times itself, or in other words, the length of any side, squared. The diagonal of a square splits it into two 45°-45°-90°, or isosceles, right triangles.

The World of Pi

You may remember being taught that the value of pi (π) is 3.14, or even 3.14159. On the GRE, π = 3ish is a close enough approximation. You don't need to be any more precise than that when doing GRE problems.

What you might not recall about pi is that pi (π) is the ratio between the circumference of a circle and its diameter. When we say that π is a little bigger than 3, we're saying that every circle is about three times as far around as it is across.

CIRCLES

Circles are a popular test topic for ETS. They have a few properties that the GRE likes to test over and over again and problems with circles also always seem to use that funny little symbol π. Here's all you need to know about circles.

Radius and Diameter

The **radius** of a circle is any line that extends from the center of the circle to the edge of the circle. If the line extends from one edge of a circle to the other and goes through the circle's center, it's the circle's **diameter**. Therefore, the diameter of a circle is twice as long as its radius.

The radius is always the key to circle problems.

Circumference of a Circle

The **circumference** of a circle is like the perimeter of a triangle: It's the distance around the outside. The formula for finding the circumference of a circle is 2 times π times the radius, or π times the diameter.

Circumference is just a fancy way of saying perimeter.

circumference = $2\pi r$ or πd

If the diameter of a circle is 4, then its circumference is 4π, or roughly 12+. If the diameter of a circle is 10, then its circumference is 10π, or a little more than 30.

An **arc** is a section of the outside, or circumference, of a circle. An angle formed by two radii is called a **central angle** (it comes out to the edge from the center of the circle). There are 360 degrees in a circle, so if there is an arc formed by, say, a 60-degree central angle, and 60 is $\frac{1}{6}$ of 360, then the arc formed by this 60-degree central angle will be $\frac{1}{6}$ of the circumference of the circle.

When working with π, leave it as π in your calculations. Also, leave $\sqrt{3}$ as $\sqrt{3}$. The answer will have them that way.

AREA OF A CIRCLE

The area of a circle is equal to π times the square of its radius.

$$\text{area} = \pi r^2$$

Let's try an example of a circle question.

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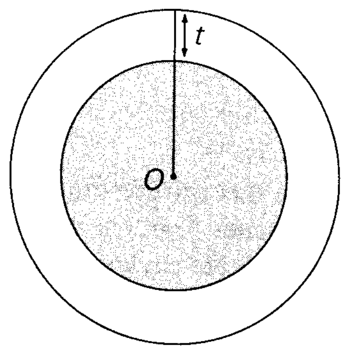

Note: Figure not drawn to scale.

In the wheel above, with center O, the area of the entire wheel is 169π. If the area of the shaded hubcap is 144π, then t =

[ ]

Enter your answer in the box above.

Here's How to Crack It

We have to figure out what t is, and it's going to be the length of the radius of the entire wheel minus the length of the radius of the hubcap. If the area of the entire wheel is 169π, the radius is $\sqrt{169}$, or 13. If the area of the hubcap is 144π, the radius is $\sqrt{144}$, or 12. $13 - 12 = 1$. Enter this value into the box.

Let's try another one.

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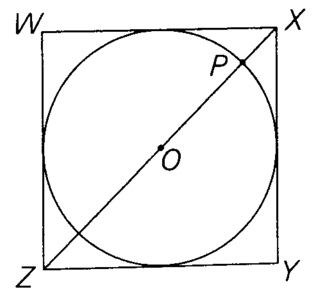


In the figure above, a circle with the center O is inscribed in square $WXYZ$. If the circle has radius 3, then PZ =

- ○ 6
- ○ $3\sqrt{2}$
- ○ $6 + \sqrt{2}$
- ○ $3 + \sqrt{3}$
- ○ $3\sqrt{2} + 3$

Ballparking answers will help you eliminate choices.

Here's How to Crack It

Inscribed means that the edges of the shapes are touching. The radius of the circle is 3, which means that PO is 3. If Z were at the other end of the diameter from P, this problem would be easy and the answer would be 6, right? But Z is beyond the edge of the circle, which means that PZ is a little more than 6. Let's stop there for a minute and glance at the answer choices. We can eliminate anything that's "out of the ballpark"—in other words, any answer choice that's less than 6, equal to 6 itself, or a lot more than 6. Remember when we told you to memorize a few of those square roots?

Let's use them:

(A) Exactly 6? Nope.

(B) That's 1.4 × 3, which is 4.2. Too small.

(C) That's 6 + 1.4, or 7.4. Not bad. Let's leave that one in.

(D) That's 3 + 1.7, or 4.7. Too small.

(E) That's (3 × 1.4) + 3, which is 4.2 + 3, or 7.2. Not bad. Let's leave that one in, too.

So we eliminated three choices with Ballparking. We're left with (C) and (E). You could take a guess here if you had to, but let's do a little more geometry to find the correct answer.

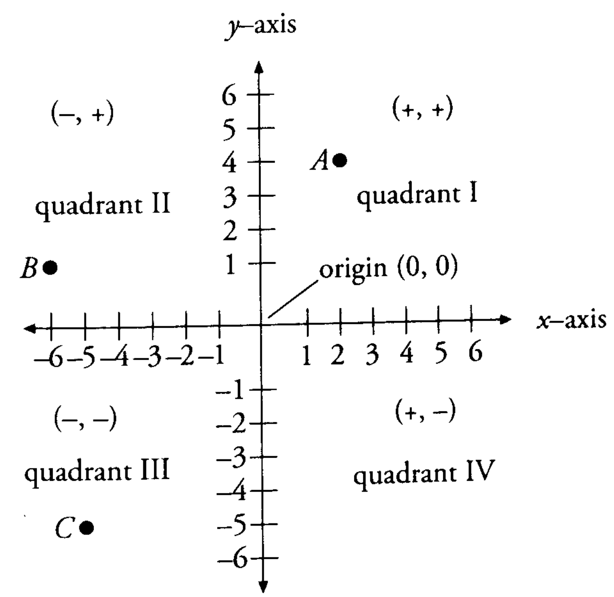
Because this circle is inscribed in the square, the diameter of the circle is the same as a side of the square. We already know that the diameter of the circle is 6, so that means that *ZY*, and indeed all the sides of the square, are also 6. Now, if *ZY* is 6, and *XY* is 6, what's *XZ*, the diagonal of the square? Well, *XZ* is also the hypotenuse of the isosceles right triangle *XYZ*. The hypotenuse of a right triangle with two sides of 6 is $6\sqrt{2}$. That's approximately 6 × 1.4, or 8.4.

The question is asking for *PZ*, which is a little less than *XZ*. It's somewhere between 6 and 8.4. The pieces that aren't part of the diameter of the circle are equal to 8.4 – 6, or 2.4. Divide that in half to get 1.2, which is the distance from the edge of the circle to *Z*. That means that *PZ* is 6 + 1.2, or 7.2. Check your remaining answers: Choice (C) is 7.4, and choice (E) is 7.2. Bingo! The answer is (E).

THE COORDINATE SYSTEM

On a coordinate system, the horizontal line is called the ***x*-axis** and the vertical line is called the ***y*-axis**. The four areas formed by the intersection of these axes are called **quadrants**. The point where the axes intersect is called the **origin**. This is what it looks like:

Coordinate geometry questions often test basic shapes such as triangles and squares.



To express any point in the coordinate system, you first give the horizontal value, then the vertical value, or (x, y). In the diagram above, point *A* can be described by the coordinates (2, 4). That is, the point is two spaces to the right of the origin and four spaces above the origin. Point *B* can be described by the coordinates (–6, 1). That is, it is six spaces to the left and one space above the origin. What are the coordinates of point *C*? Right, it's (–5, –5).

ALWAYS write A, B, C, D, E on your scratch paper to represent the answer choices (or A, B, C, D if it's a quant comp question.)

Here's a GRE example:

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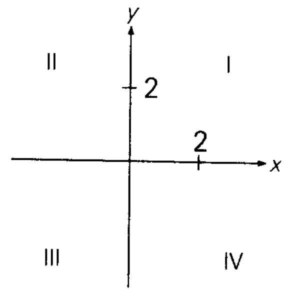

Points $(x, 5)$ and $(-6, y)$, not shown in the figure above, are in quadrants I and III, respectively. If $xy \neq 0$, in which quadrant is point (x, y) ?

- ○ IV
- ○ III
- ○ II
- ○ I
- ○ It cannot be determined from the information given.

Here's How to Crack It

If point $(x, 5)$ is in quadrant I, that means x is positive. If point y is in quadrant III, then y is negative. The quadrant that would contain coordinate points with a positive x and a negative y is quadrant IV. That's answer choice (A).

Slope

Trickier questions involving the coordinate system might give you the equation for a line on the grid, which will involve something called the slope of the line. The equation of a line is

$$y = mx + b$$

In this equation x and y are both points on the line, b stands for the y-intercept, or the point at which the line crosses the y-axis, and m is the slope of the line.

Slope is defined as the vertical change divided by the horizontal change, often called "the rise over the run" or the "change in y over the change in x."

$$\text{Slope} = \frac{\text{rise}}{\text{run}}$$

Sometimes on the GRE, m is written instead as a, as in $y = ax + b$. You'll see all this in action in a moment.

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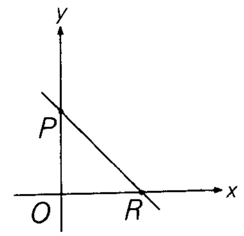

The line $y = -\frac{8}{7}x + 1$ is graphed on the rectangular coordinate axes.

Quantity A	Quantity B
OR	OP

- ○ Quantity A is greater.
- ○ Quantity B is greater.
- ○ The two quantities are equal.
- ○ The relationship cannot be determined from the information given.

Here's How to Crack It

The y-intercept, or b, in this case is 1. That means the line crosses the y-axis at 1. So the coordinates of point P are (0, 1). Now we have to figure out what the coordinates of point R are. We know the y-coordinate is 0, so let's stick that into the equation (the slope and the y-intercept are constant; they don't change).

$$y = mx + b$$

$$0 = -\frac{8}{7}x + 1$$

Now let's solve for x.

$$0 = -\frac{8}{7}x + 1$$

$$0 - 1 = -\frac{8}{7}x + 1 - 1$$

$$-1 = -\frac{8}{7}x$$

$$\left(-\frac{7}{8}\right)(-1) = \left(-\frac{7}{8}\right)\left(-\frac{8}{7}\right)x$$

$$\frac{7}{8} = x$$

So the coordinates of point R are $(\frac{7}{8}, 0)$. That means OR, in Quantity A, is equal to $\frac{7}{8}$, and OP, in Quantity B, is equal to 1. The answer is (B).

Another approach to this question would be to focus on the meaning of slope. Because the slope is $-\frac{8}{7}$, that means the vertical change is 8 and the horizontal change is 7. In other words, you count up 8 and over 7. Clearly the "up" is more than the "over," thus OP is more than OR.

Incidentally, if you're curious about the difference between a positive and negative slope, any line that rises from left to right has a positive slope. Any line that falls from left to right has a negative slope. (A horizontal line has a slope of 0, and a vertical line is said to have "no slope.")

VOLUME

You can find the volume of a three-dimensional figure by multiplying the area of a two-dimensional figure by its height (or depth). For example, to find the volume of a rectangular solid, you would take the area of a rectangle and multiply it by the depth. The formula is lwh (length × width × height). To find the volume of a circular cylinder, take the area of a circle and multiply by the height. The formula is πr^2 times the height (or $\pi r^2 h$).

DIAGONALS IN THREE DIMENSIONS

There's a special formula that you can use if you are ever asked to find the length of a diagonal (the longest distance between any two corners) inside a three-dimensional rectangular box. It is $a^2 + b^2 + c^2 = d^2$, where a, b, and c are the dimensions of the figure (kind of looks like the Pythagorean theorem, huh?).

Questions that ask about diagonals are really about the Pythagorean theorem.

Take a look:

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What is the length of the longest distance between any two corners in a rectangular box with dimensions 3 inches by 4 inches by 5 inches?

- ○ 5
- ○ 12
- ○ $5\sqrt{2}$
- ○ $12\sqrt{2}$
- ○ 50

Here's How to Crack It

Let's use our formula, $a^2 + b^2 + c^2 = d^2$. The dimensions of the box are 3, 4, and 5.

$$3^2 + 4^2 + 5^2 = d^2$$
$$9 + 16 + 25 = d^2$$
$$50 = d^2$$
$$\sqrt{50} = d$$
$$\sqrt{25 \times 2} = d$$
$$\sqrt{25} \times \sqrt{2} = d$$
$$5\sqrt{2} = d$$

That's choice (C).

Don't confuse surface area with volume.

SURFACE AREA

The surface area of a rectangular box is equal to the sum of the areas of all of its sides. In other words, if you had a box whose dimensions were 2 × 3 × 4, there would be two sides that are 2 by 3 (this surface would have an area of 6), two sides that are 3 by 4 (area of 12), and two sides that are 2 by 4 (area of 8). So, the total surface area would be 6 + 6 + 12 + 12 + 8 + 8, which is 52.

Key Formulas and Rules

Here is a review of the key rules and formulas to know for the GRE Math section.

Lines and angles

- All straight lines have 180 degrees.
- A right angle measures 90 degrees.
- Vertical angles are equal.
- Parallel lines cut by a third lines have two angles, big angles and small angles. All of the big angles are equal and all of the small angles are equal. The sum of a big angle and a small angle is 180 degrees.

Triangles

- All triangles have 180 degrees.
- The angles and sides of a triangle are in proportion—the largest angle is opposite the largest side and the smallest side is opposite the smallest angle.
- The Pythagorean theorem is $c^2 = a^2 + b^2$ where c is the length of the hypotenuse.
- The area formula for a triangle is $A = \frac{bh}{2}$.

Quadrilaterals

- All quadrilaterals have 360 degrees.
- The area formula for a squares and rectangles is bh.

Circles

- All circles have 360 degrees.
- The radius is the distance from the center of the circle to any point on the edge.
- The area of a circle is πr^2.
- The perimeter of a circle is $2\pi r$.

PLUGGING IN ON GEOMETRY PROBLEMS

Remember, whenever you have a question that has answer choices, like a regular multiple choice or a multiple choice, multiple answer question that has variables in the answer choices, Plug In. On geometry problems, you can plug in values for angles or lengths as long as the values you plug in don't contradict either the wording of the problem or the laws of geometry (you can't have the interior angles of a triangle add up to anything but 180, for instance).

Here's an example:

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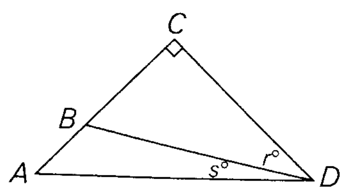

In the drawing above, if $AC = CD$, then $r =$

- ○ $45 - s$
- ○ $90 - s$
- ○ s
- ○ $45 + s$
- ○ $60 + s$

Here's How to Crack It

See the variables in the answer choices? Let's Plug In. First of all, we're told that AC and CD are equal, which means that ACD is an isosceles right triangle. So angles A and D both have to be 45 degrees. Now it's Plugging In time. The smaller angles, r and s, must add up to 45 degrees, so let's make $r = 40$ degrees and $s = 5$ degrees. The question asks for the value of r, which is 40, so that's our target answer. Now eliminate answer choices by plugging in 5 for s .

Don't forget to Plug In on geometry questions. Just pick numbers according to the rules of geometry

(A) 45 − 5 = 40. Bingo! Check the other choices to be sure.

(B) 90 − 5 = 85. Nope.

(C) 5. Nope.

(D) 45 + 5 = 50. Eliminate it.

(E) 60 + 5 = 65. No way.

By the way, we knew that the correct answer couldn't be greater than 45 degrees, because that's the measure of the entire angle D, so you could have eliminated (D) and (E) right away.

DRAW IT YOURSELF

When ETS doesn't include a drawing with a geometry problem, it usually means that the drawing, if supplied, would make ETS's answer obvious. In cases like this, you should just draw it yourself. Here's an example:

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Quantity A	Quantity B
The diameter of a circle with area 49π	14

- ○ Quantity A is greater.
- ○ Quantity B is greater.
- ○ The two quantities are equal.
- ○ The relationship cannot be determined from the information given.

Here's How to Crack It

Visualizing the figure, if the area is 49π, what's the radius? Right: 7. And if the radius is 7, what's the diameter? Right: 14. The answer is (C).

Redraw

On tricky quant comp questions, you may need to draw the figure once, eliminate two answer choices, and then draw it another way to try to disprove your first answer; in order to see if the answer is (D). Here's an example of a problem that might require you to do this:

For quant comp geometry questions, draw, eliminate, and REDRAW; it's like Plugging In twice.

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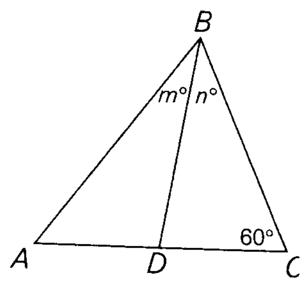


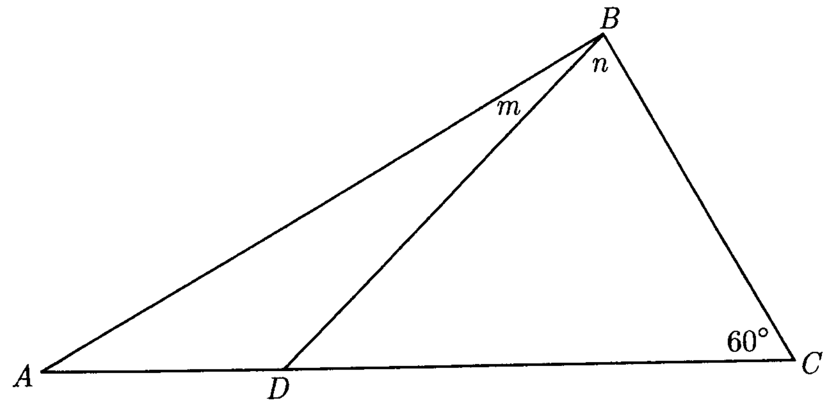
D is the midpoint of AC.

Quantity A	Quantity B
m	n

- Quantity A is greater.
- Quantity B is greater.
- The two quantities are equal.
- The relationship cannot be determined from the information given.

Here's How to Crack It

Are you sure that the triangle looks exactly like this? Nope. We only know what it tells us—that the lengths of AD and DC are equal; from this figure, it looks like angles m and n are also equal. Because this means that it's possible for them to be, we can eliminate choices (A) and (B). But let's redraw the figure to try to disprove our first answer.



Try drawing the triangle as stretched out as possible. Notice that n is now clearly greater than m, so you can eliminate (C), and the answer is (D).

Geometry Drill

Think you've mastered these concepts? Try your hand at the following problems and check your work after you've finished. You can find the answers in Part V.

1 of 15

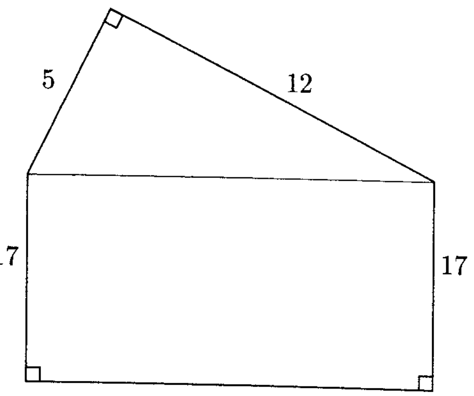
Which of the following could be the degree measures of two angles in a right triangle?

*Indicate **all** possible values.*

- ☐ 20° and 70°
- ☐ 30° and 60°
- ☐ 45° and 45°
- ☐ 55° and 55°
- ☐ 75° and 75°

Click on your choice(s).

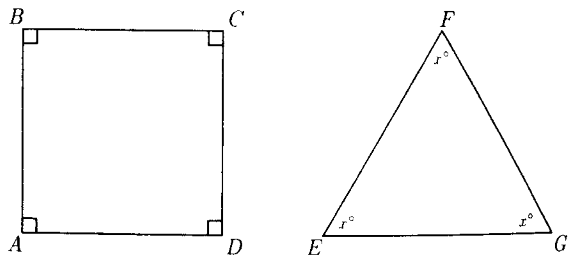
2 of 15



What is the perimeter of the figure above?

- ○ 51
- ○ 64
- ○ 68
- ○ 77
- ○ 91

3 of 15



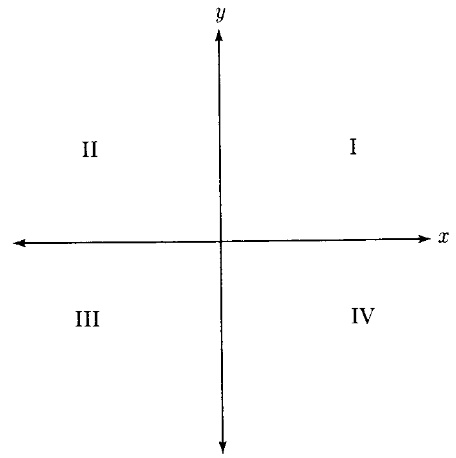
$AB = BC = EG$

$FG = 8$

Quantity A	**Quantity B**
The area of square $ABCD$	32

- ○ Quantity A is greater.
- ○ Quantity B is greater.
- ○ The two quantities are equal.
- ○ The relationship cannot be determined from the information given.

4 of 15



$(a, 6)$ is a point (not shown) in Region I.

$(-6, b)$ is a point (not shown) in Region II.

Quantity A	**Quantity B**
a	b

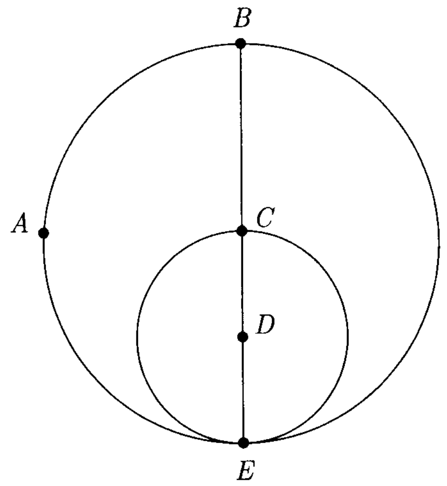
- ○ Quantity A is greater.
- ○ Quantity B is greater.
- ○ The two quantities are equal.
- ○ The relationship cannot be determined from the information given.

5 of 15

A piece of twine of length t is cut into two pieces. The length of the longer piece is 2 yards greater than 3 times the length of the shorter piece. Which of the following is the length, in yards, of the longer piece?

- ○ $\dfrac{t+3}{3}$
- ○ $\dfrac{3t+2}{3}$
- ○ $\dfrac{t-2}{4}$
- ○ $\dfrac{3t+4}{4}$
- ○ $\dfrac{3t+2}{4}$

6 of 15



A circle with center D is drawn inside in a circle with center C, as shown. If $CD = 3$, what is the area of semicircle EAB ?

- ○ $\dfrac{9}{2}\pi$
- ○ 9π
- ○ 12π
- ○ 18π
- ○ 36π

7 of 15

For the final exam in a scuba diving certification course, Karl has to navigate underwater from one point in a lake to another. Karl began the test at the boat and swam due south for 7 meters. He then turned due east and swam for x meters. When he surfaced, he was 25 meters from the boat. What is the value of x ?

☐ meters

Click on the answer box, then type in a number. Backspace to erase.